Gravitational Waves Induced by non-Gaussian Scalar Perturbations

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Rong-gen Cai, SP and Misao Sasaki,
arXiv:1810.11000, accepted by PRL;
arXiv:1906.XXXXX, in preparation.

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Content

- Mechanism of Stochastic Background GWs
- Primordial Black Holes as dark matter
- Induced GWs: a probe of PBH abundance
- Conclusion

Content

- Mechanism of Stochastic Background GWs
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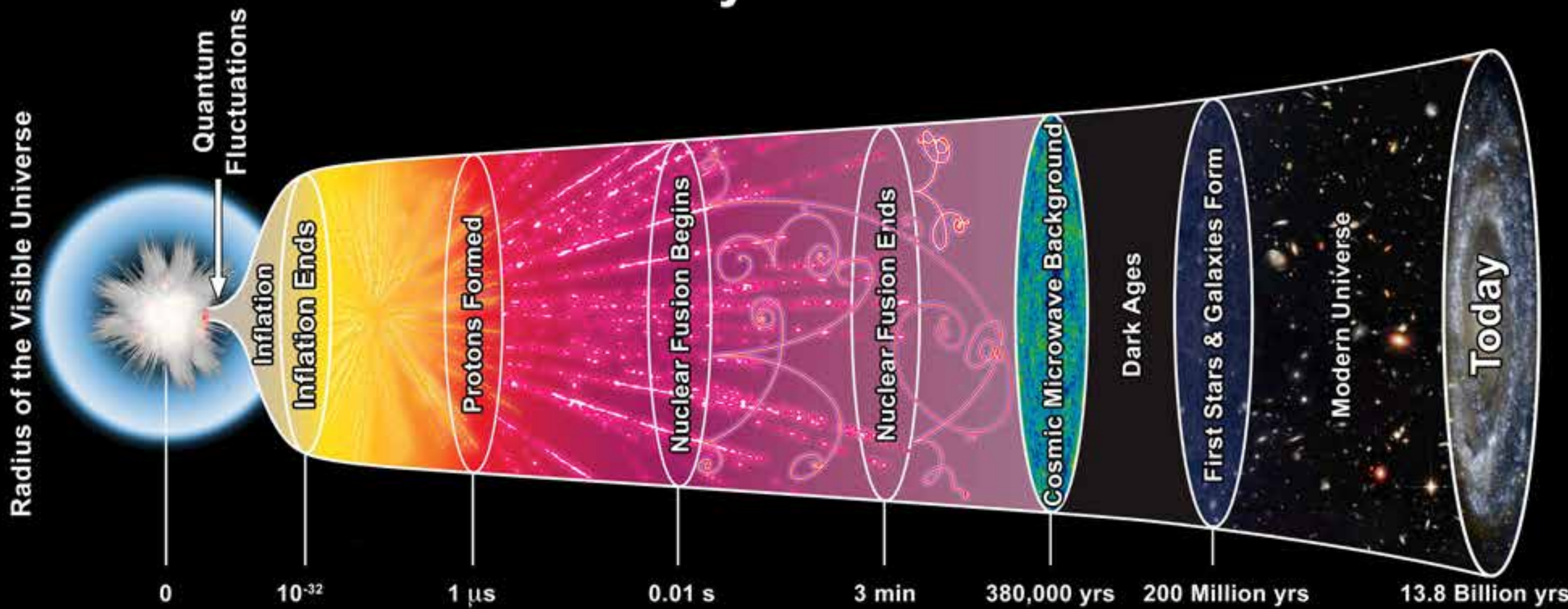
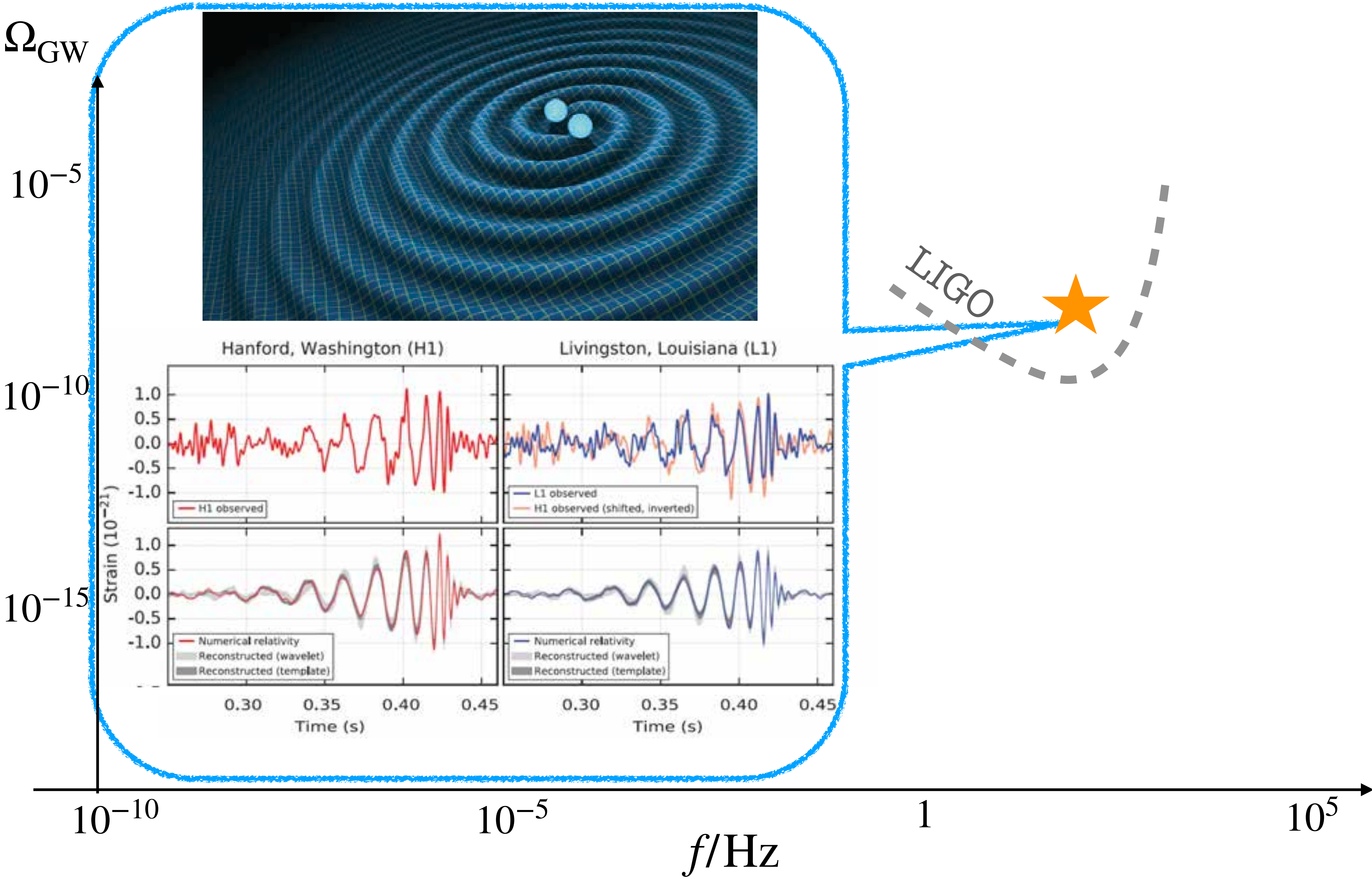
- Induced GWs: a probe of PBH abundance
- Conclusion

History of the Universe

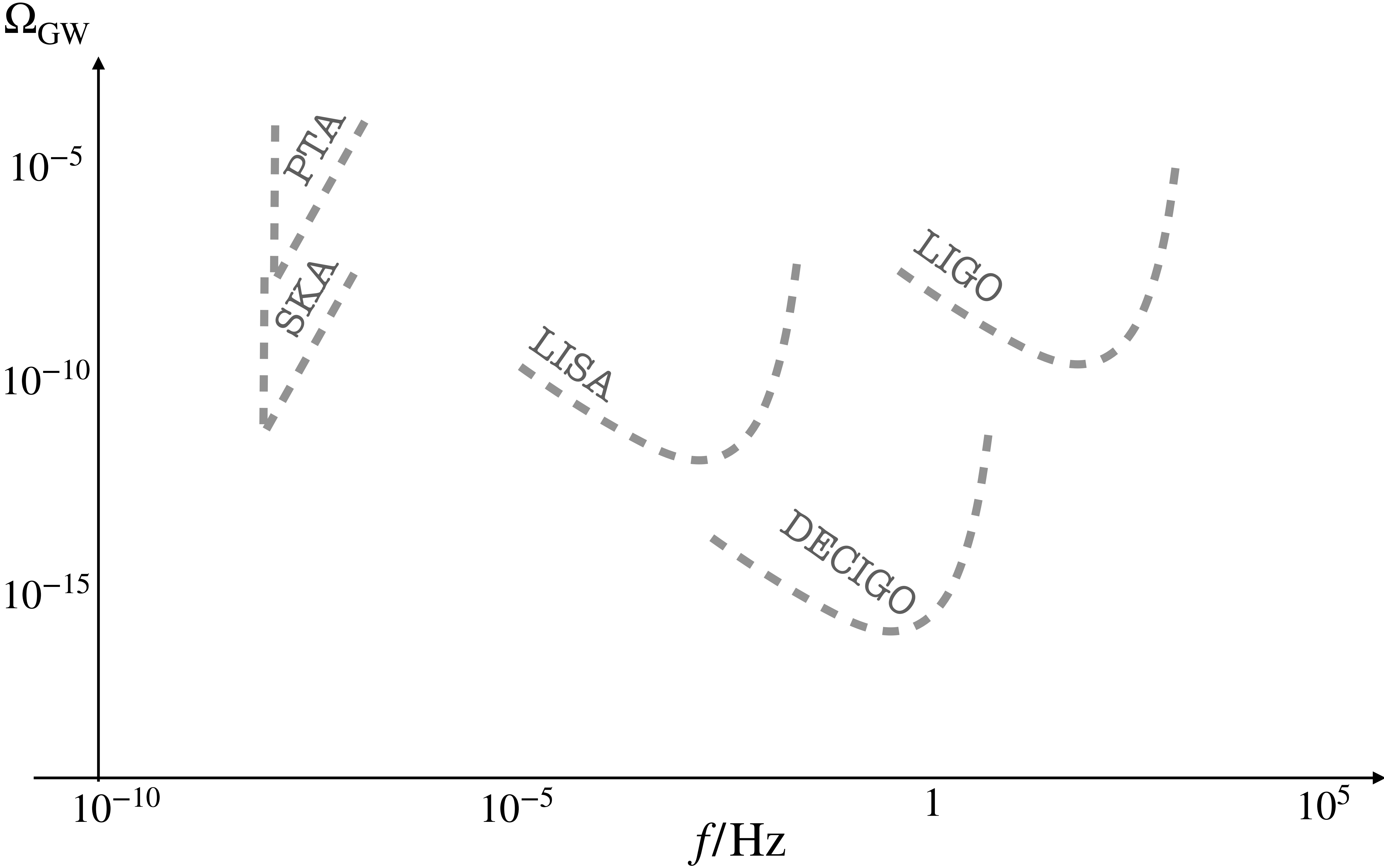


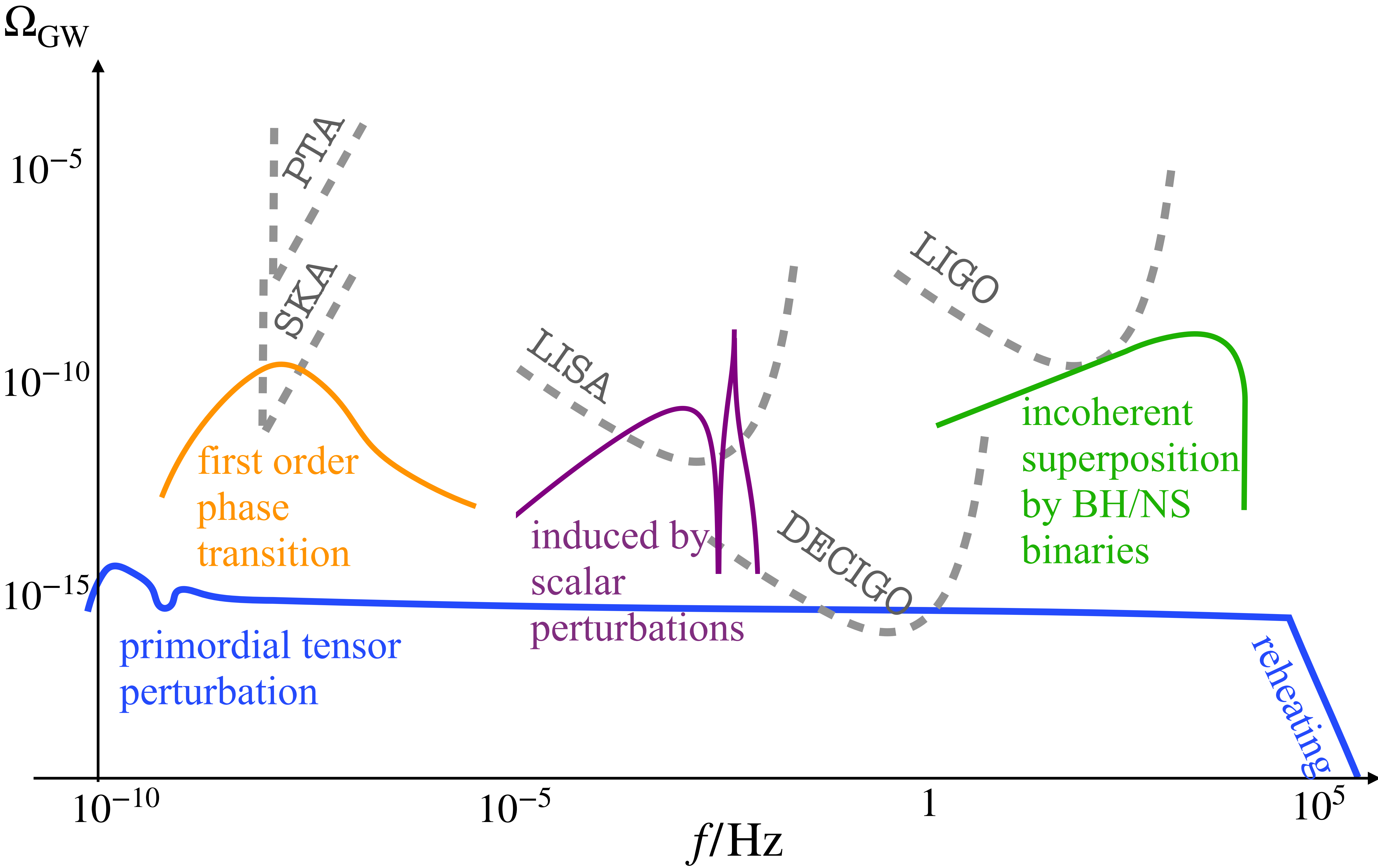
Age of the Universe

BICEP2 Collaboration/CERN/NASA

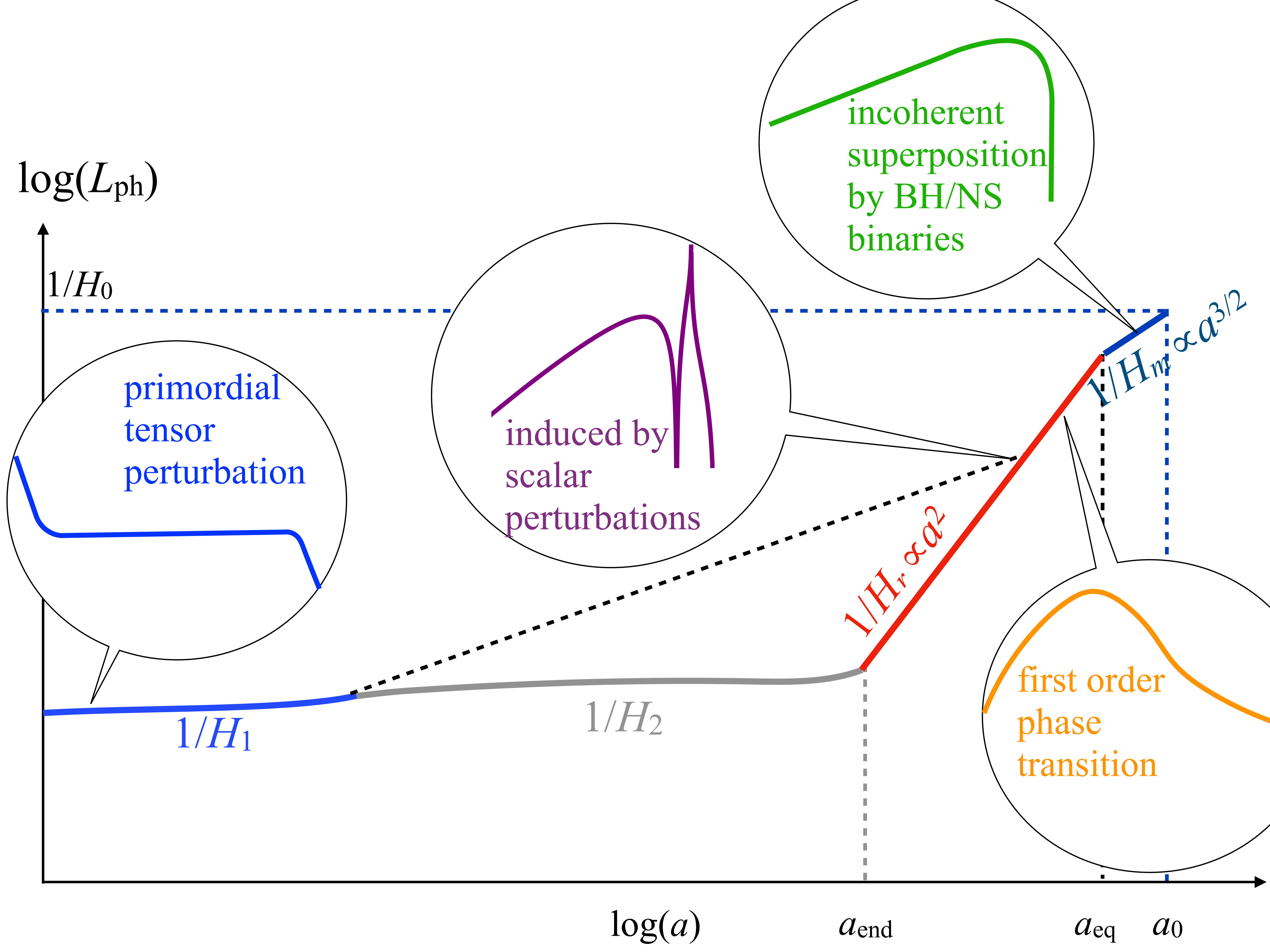


Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
10^{-10}
10^{-5}
1
10^{5}
f/Hz
LIGO
Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Time (s)

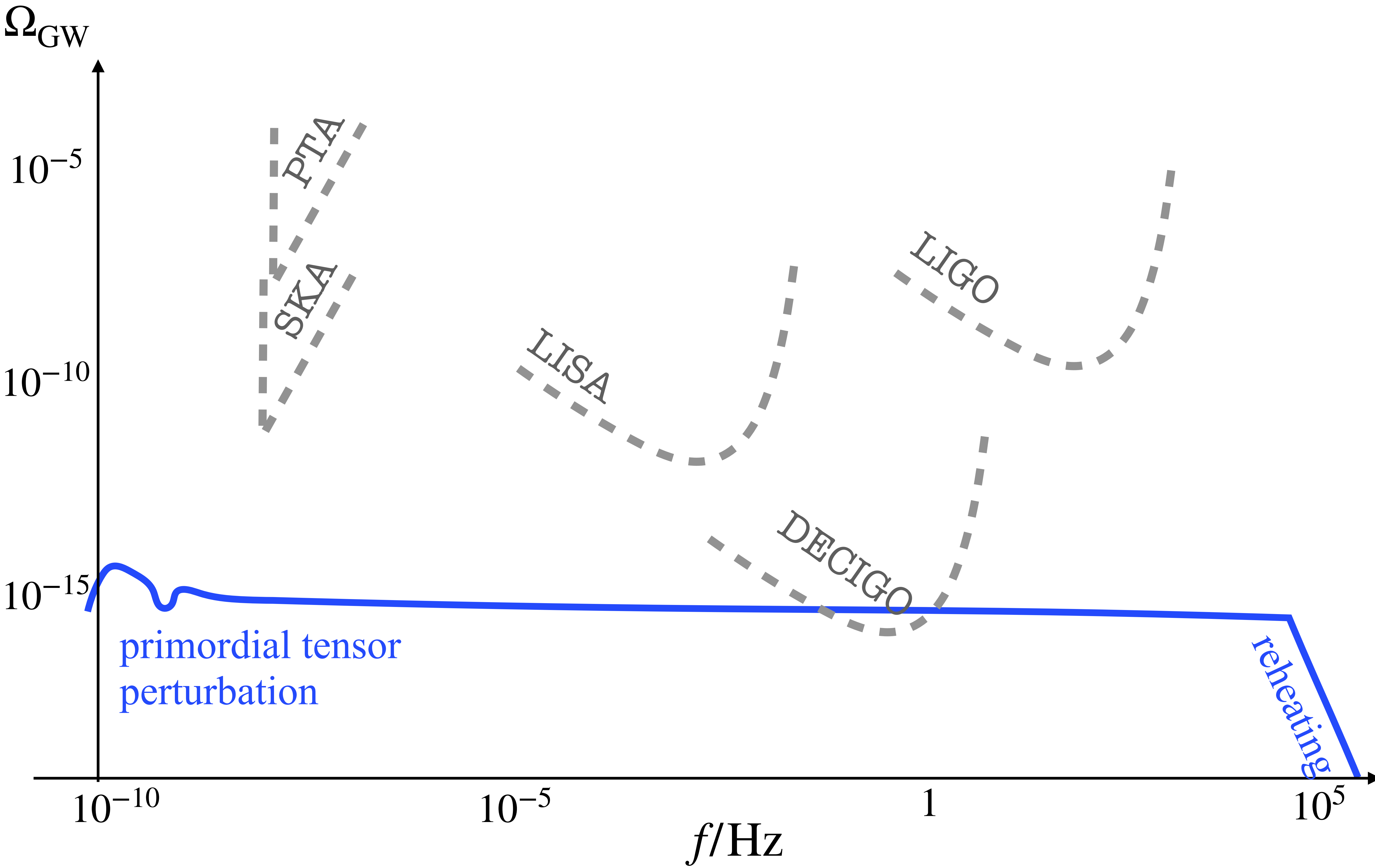

Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
10^{-10}
10^{-5}
1
10^{5}
f/Hz



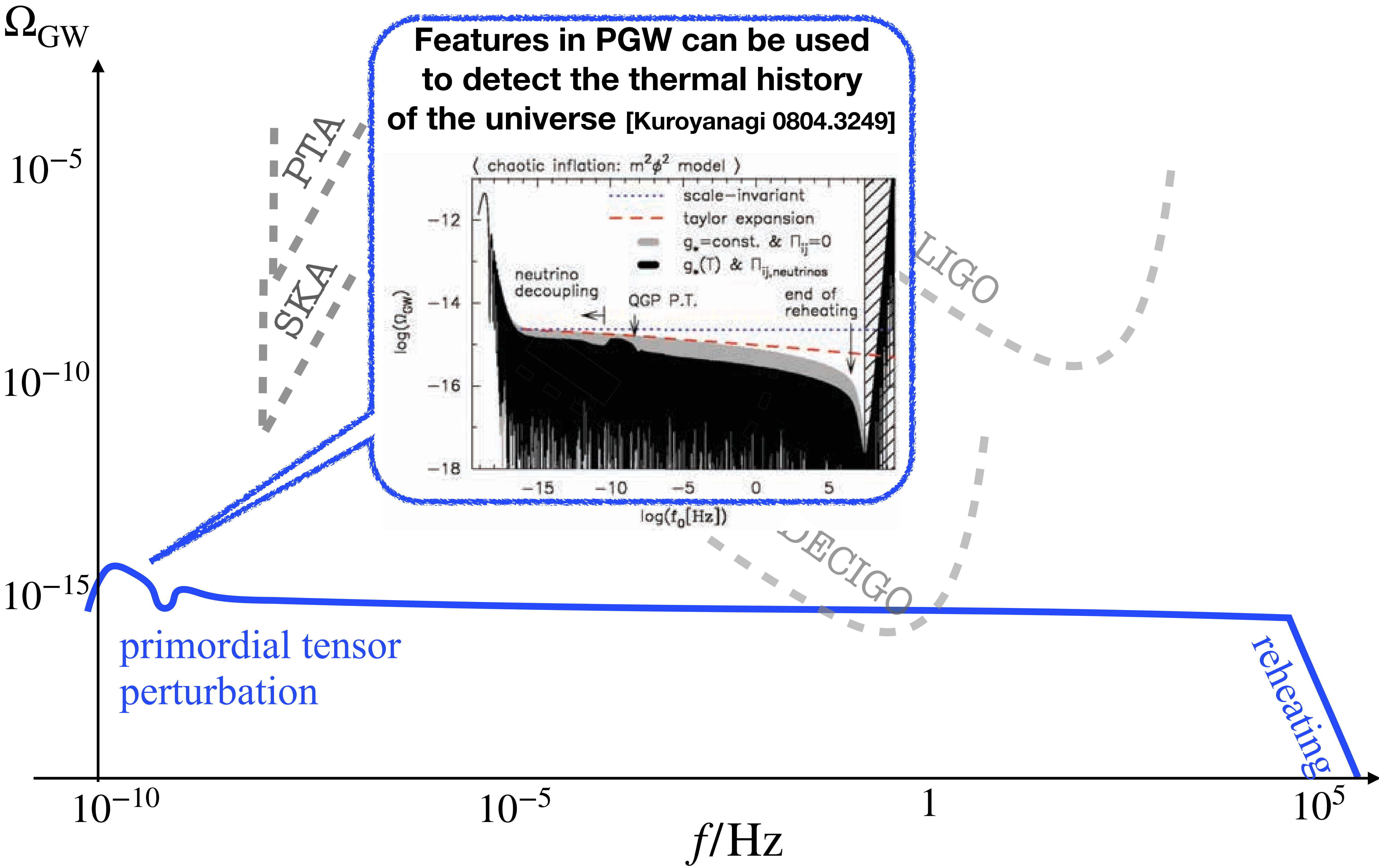
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
first order phase transition
induced by scalar perturbations
incoherent superposition by BH/NS binaries
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz



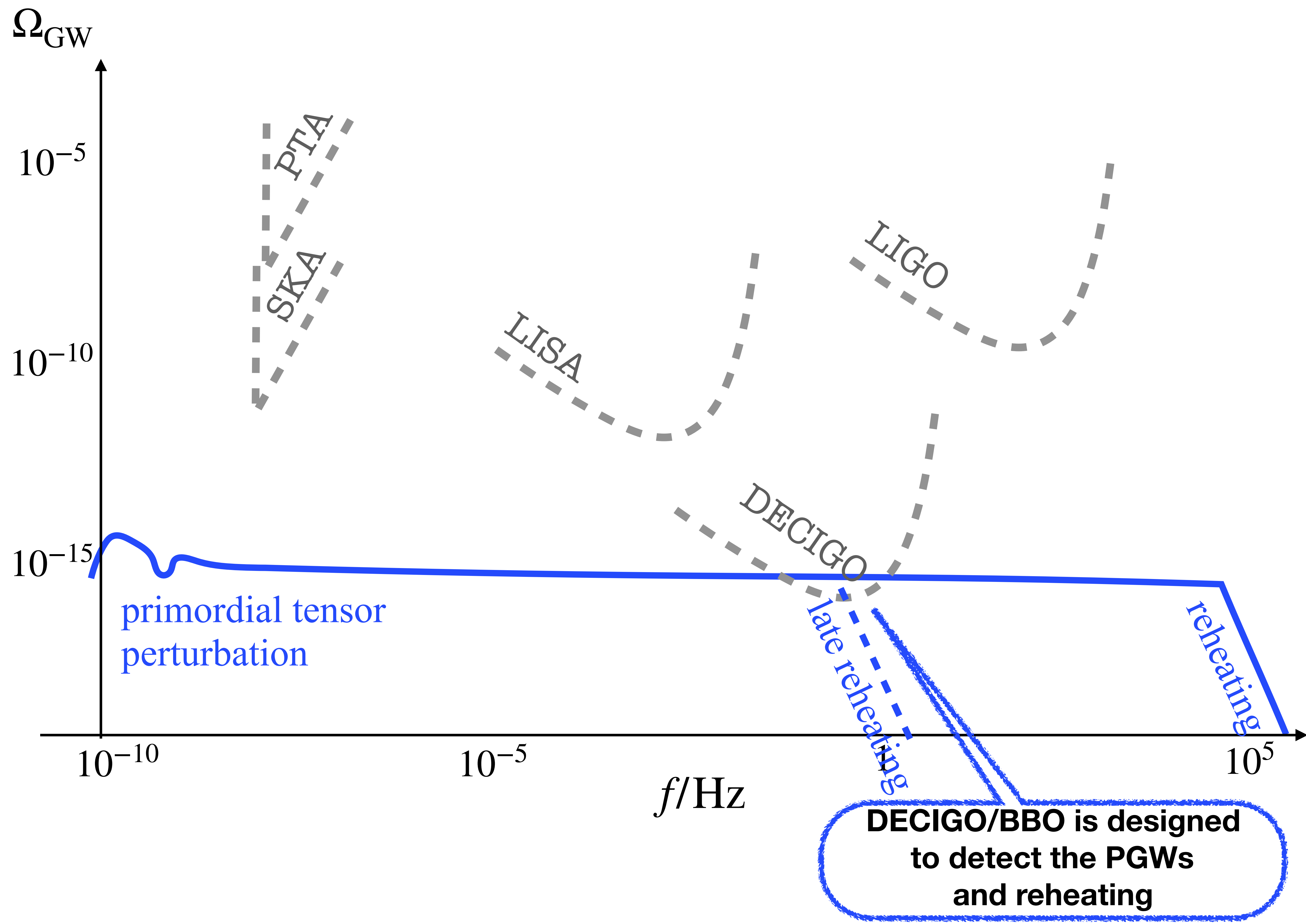
log(L_ph)
1/H_0
primordial tensor perturbation
induced by scalar perturbations
incoherent superposition by BH/NS binaries
1/H_m ∝ a^{3/2}
1/H_r ∝ a^2
first order phase transition
1/H_1
1/H_2
log(a)
a_end
a_eq
a_0



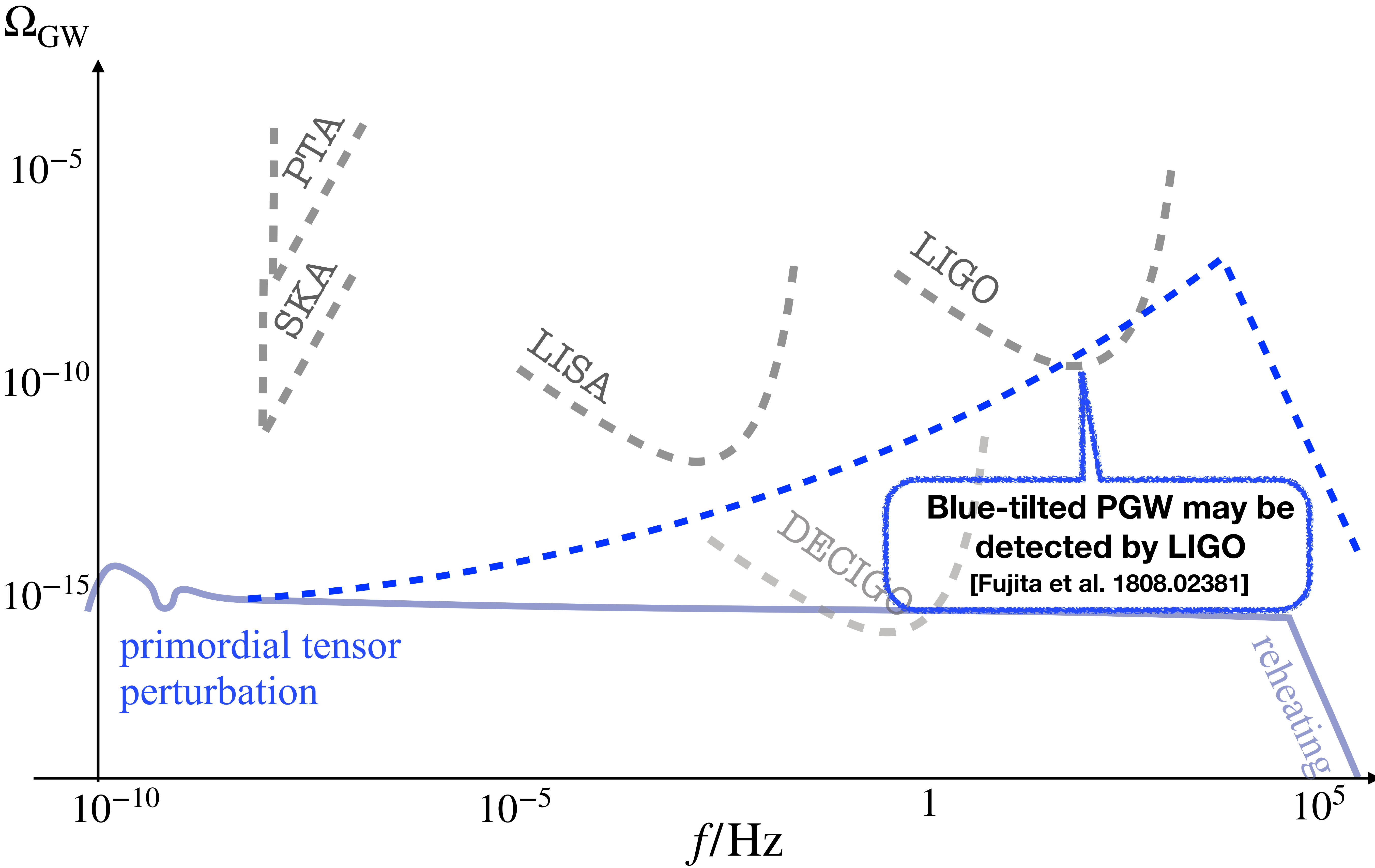
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz



Ω_{GW}
Features in PGW can be used to detect the thermal history of the universe [Kuroyanagi 0804.3249]
(chaotic inflation: $m^2\phi^2$ model)
scale−invariant
taylor expansion
g_*=const. & Π_{ij}=0
$g_*(T)$ & $\Pi_{ij,neutrinos}$
neutrino decoupling
QGP P.T.
end of reheating
log(Ω_{GW})
log(f_0[Hz])
−12
−14
−16
−18
−15
−10
−5
0
5
PTA
SKA
LIGO
DECIGO
10^{-5}
10^{-10}
10^{-15}
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz

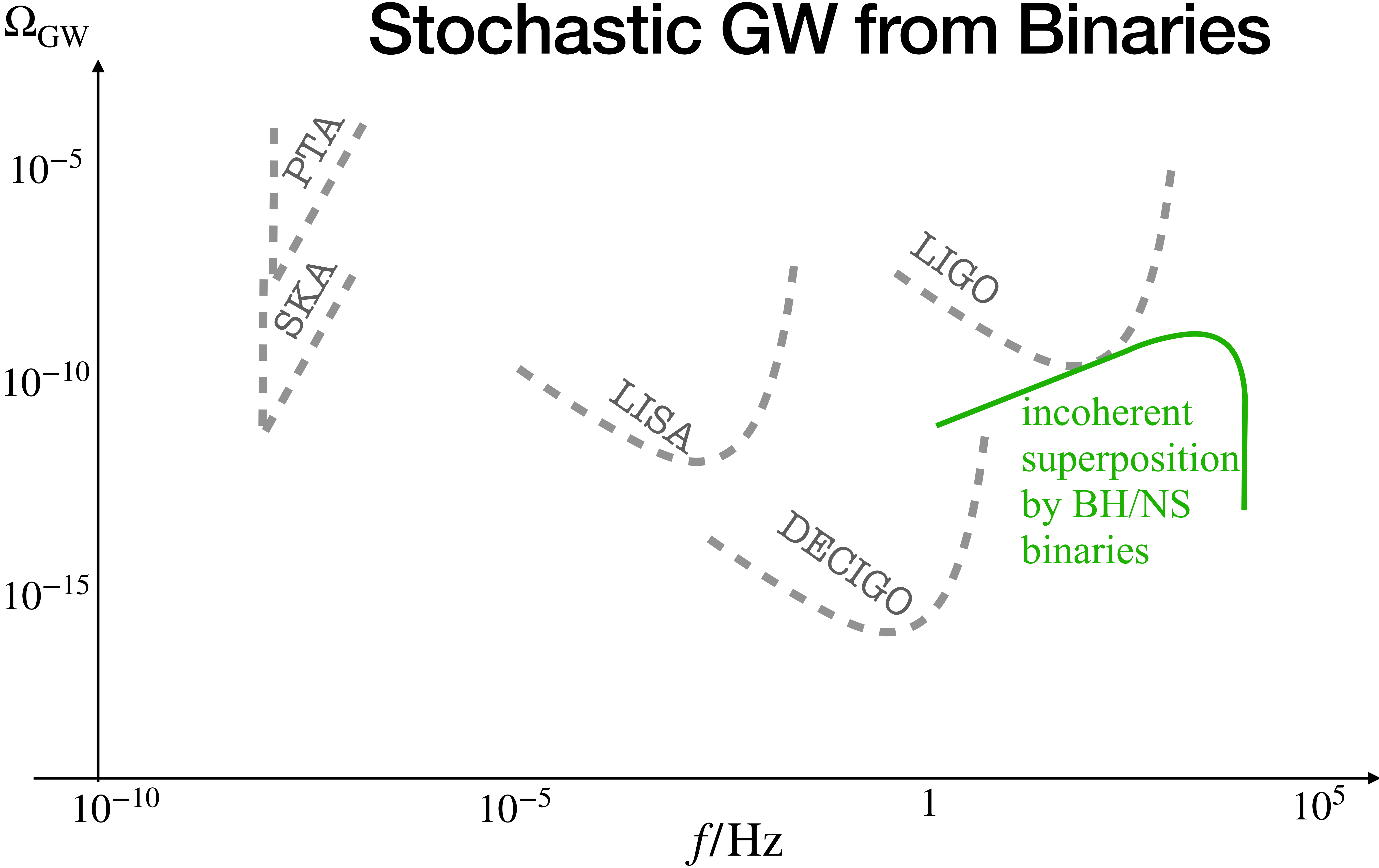


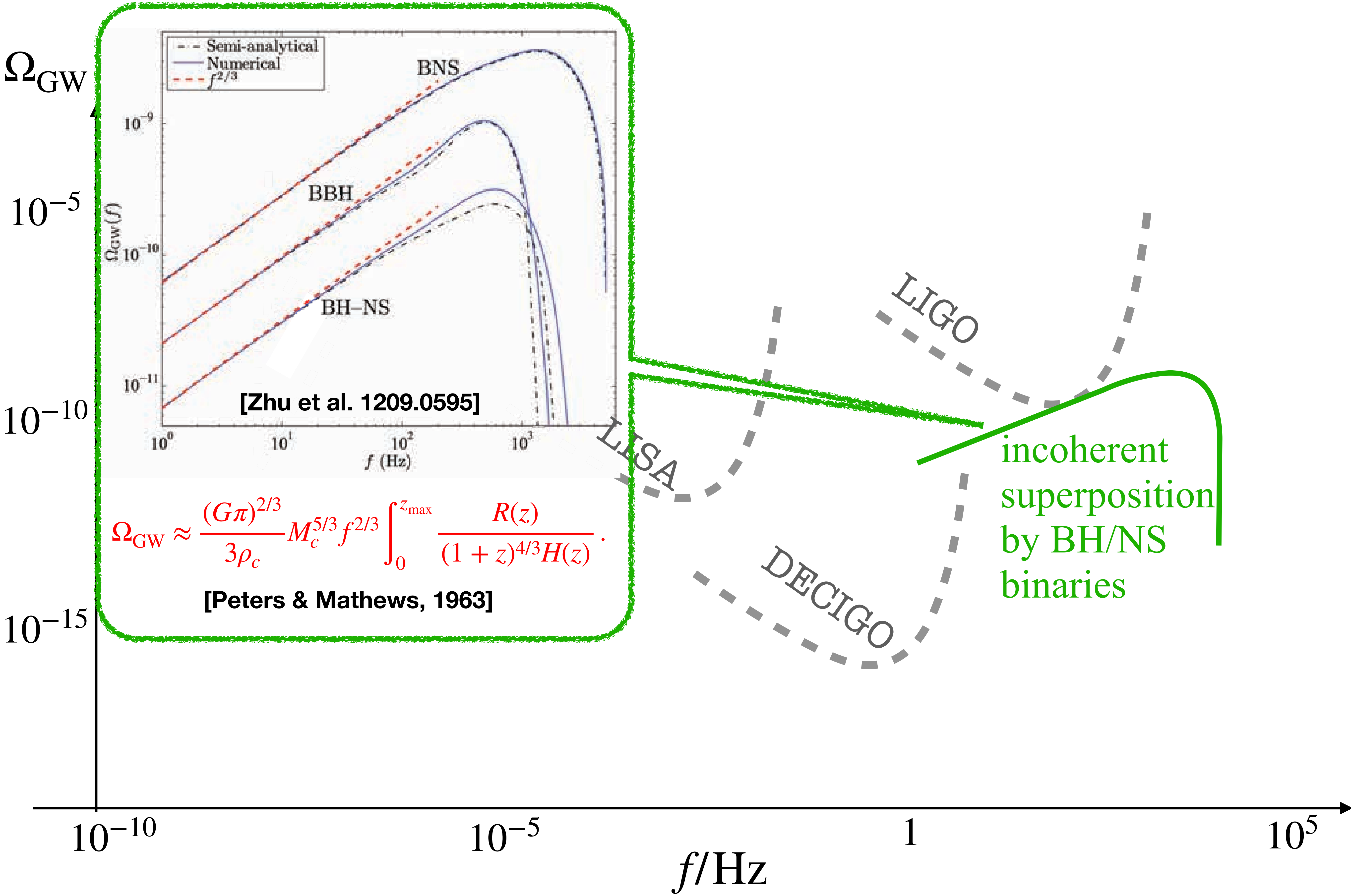
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
primordial tensor perturbation
late reheating
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz
DECIGO/BBO is designed to detect the PGWs and reheating



Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
Blue-tilted PGW may be detected by LIGO
[Fujita et al. 1808.02381]
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz

Stochastic GW from Binaries



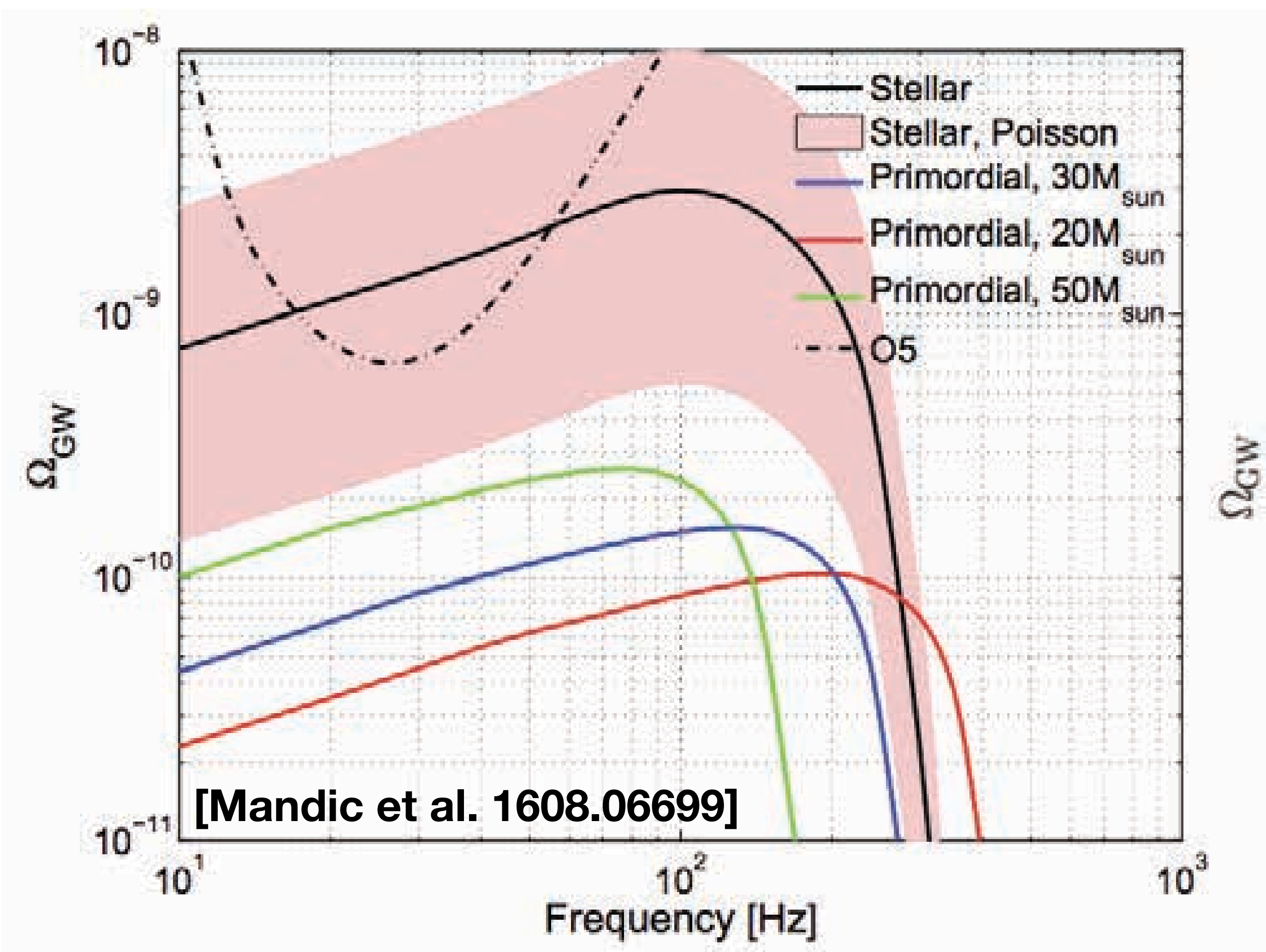


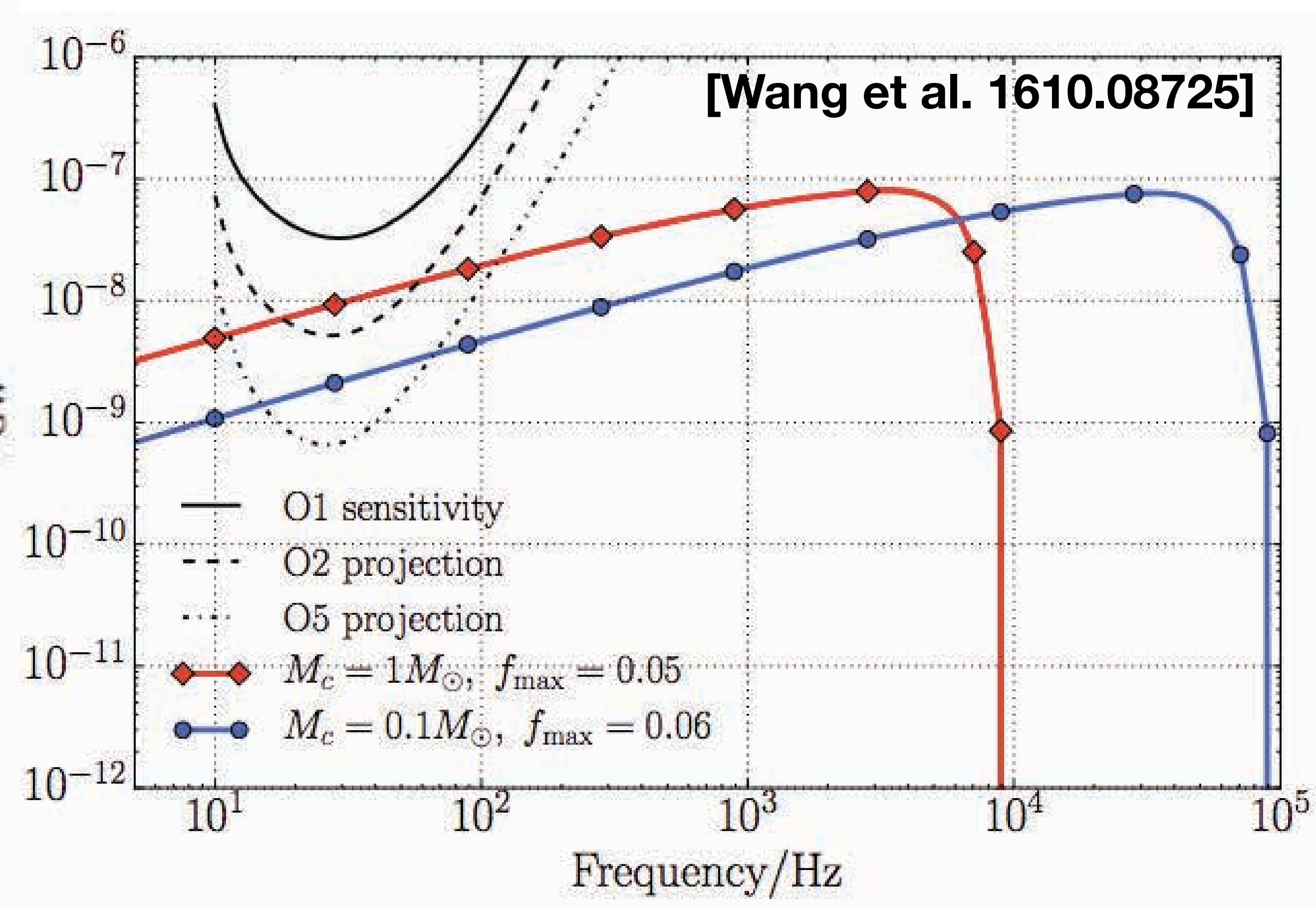
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
Semi-analytical
Numerical
$f^{2/3}$
BNS
BBH
BH–NS
$\Omega_{GW}(f)$
10^{-9}
10^{-10}
10^{-11}
10^{0}
10^{1}
10^{2}
10^{3}
f (Hz)
[Zhu et al. 1209.0595]
$\Omega_{GW} \approx \frac{(G\pi)^{2/3}}{3\rho_c} M_c^{5/3} f^{2/3} \int_0^{z_{max}} \frac{R(z)}{(1+z)^{4/3} H(z)}$.
[Peters & Mathews, 1963]
LIGO
LISA
DECIGO
incoherent superposition by BH/NS binaries
10^{-10}
10^{-5}
1
10^{5}
f/Hz

Stochastic GW from Binaries

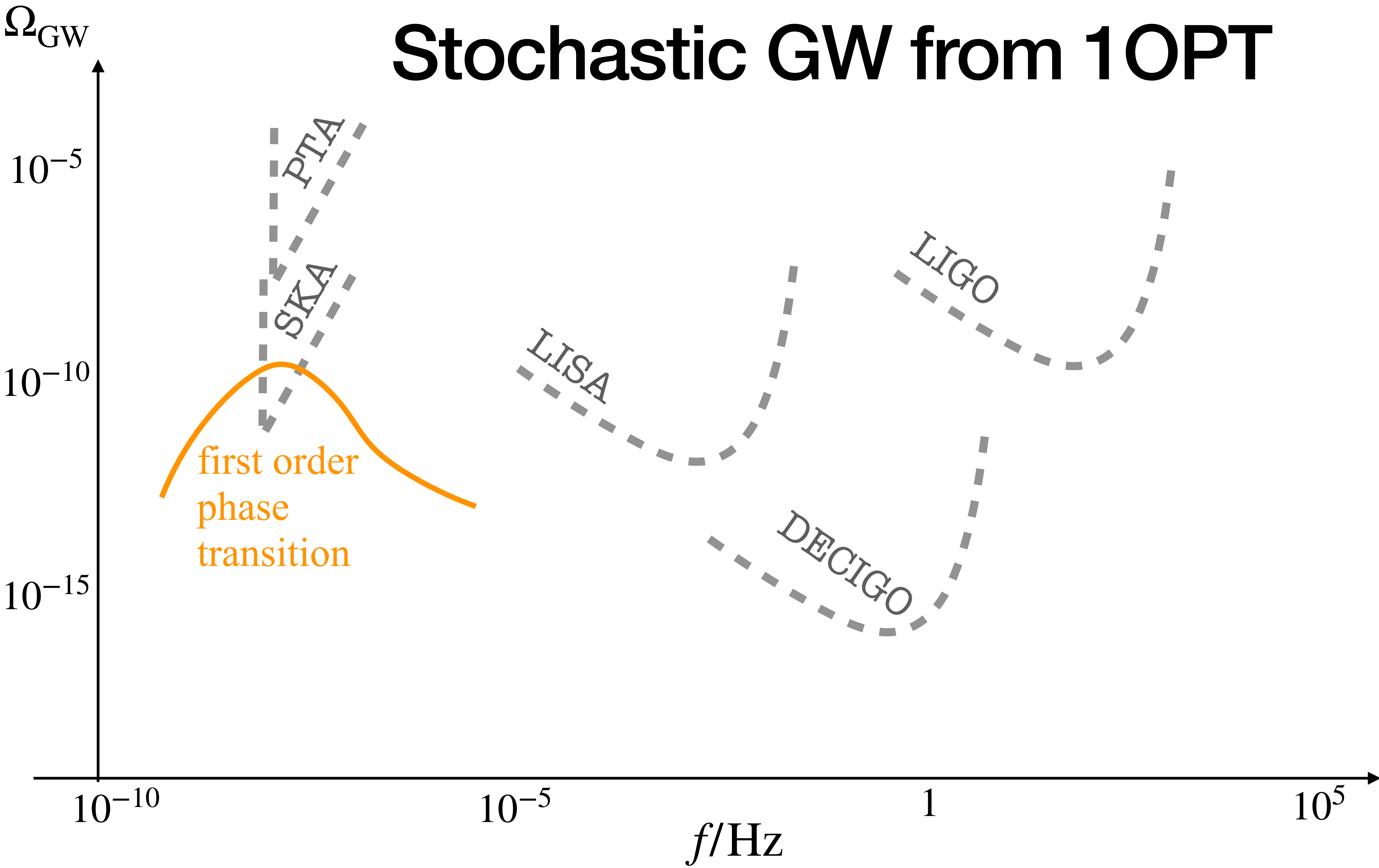
- Origin: incoherent superposition of the GWs emitted by BH(NS) binaries
- Frequencies: LIGO
- Amplitude: 10^{-9}

$$\Omega_{\rm GW} = \frac{f}{\rho_c}\int_0^{z_{\max}} dz \frac{R(z)}{(1+z)H(z)} \left(\frac{dE_{\rm gw}}{df}(f_r)\right)_{f_r=(1+z)f} .$$





Stochastic GW from 1OPT

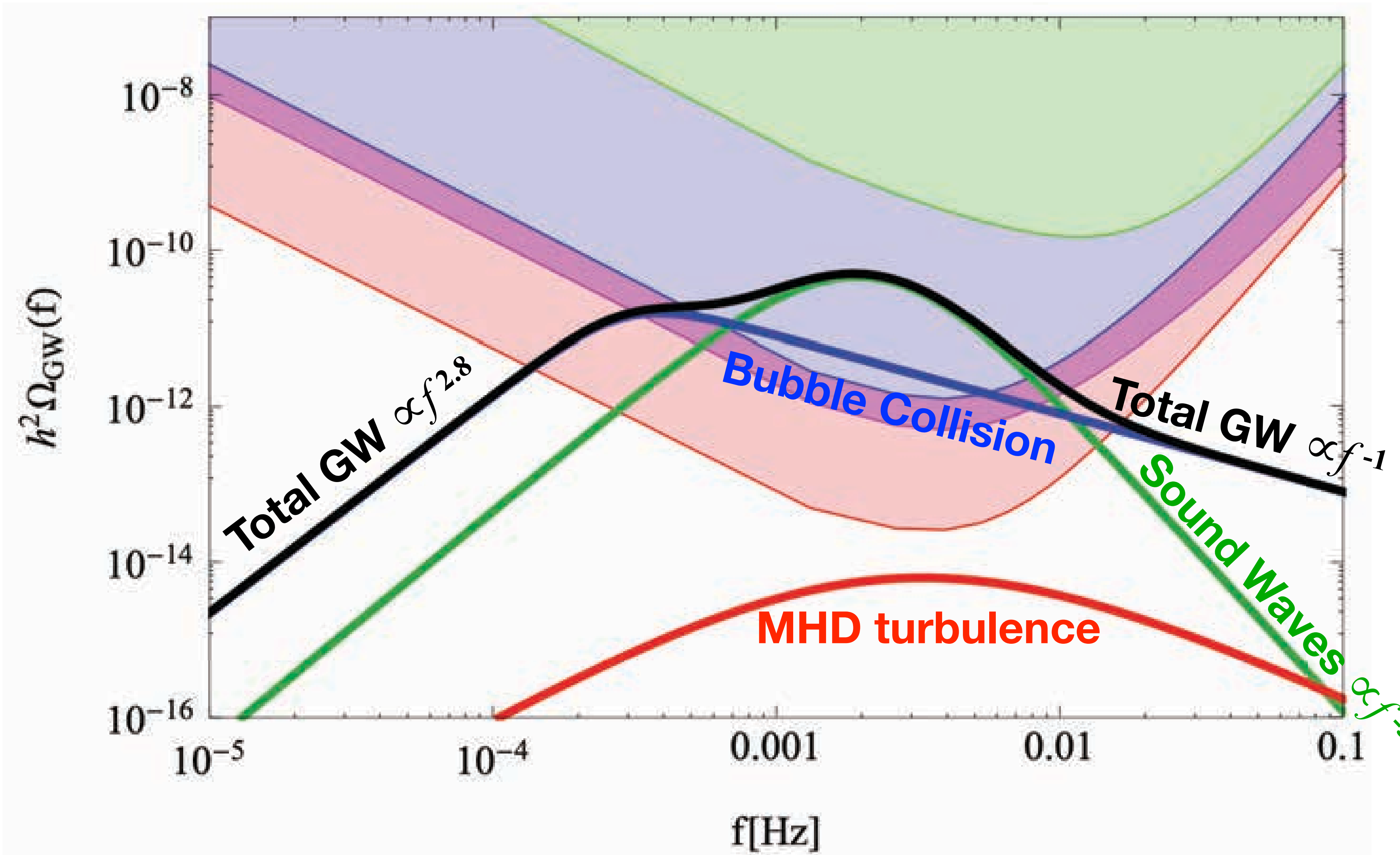


Stochastic GW from 1OPT

$$f_{\rm peak} \simeq 10^{-6}{\rm Hz} \left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{100{\rm GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} \qquad \Omega_{\rm peak}h^2 \simeq 10^{-6} \left(\frac{\beta}{H_*}\right)^{-2} \left(\frac{g_*}{100}\right)^{-1/3}$$

- Key feature: k^3 increasing, k^{-2} or k^{-1} decreasing.
- For β/H_*~100, frequency is 10^{-3}Hz, in LISA band. It is possible to detect its peak and ultraviolet tail.

[Caprini et al. 1512.06239]



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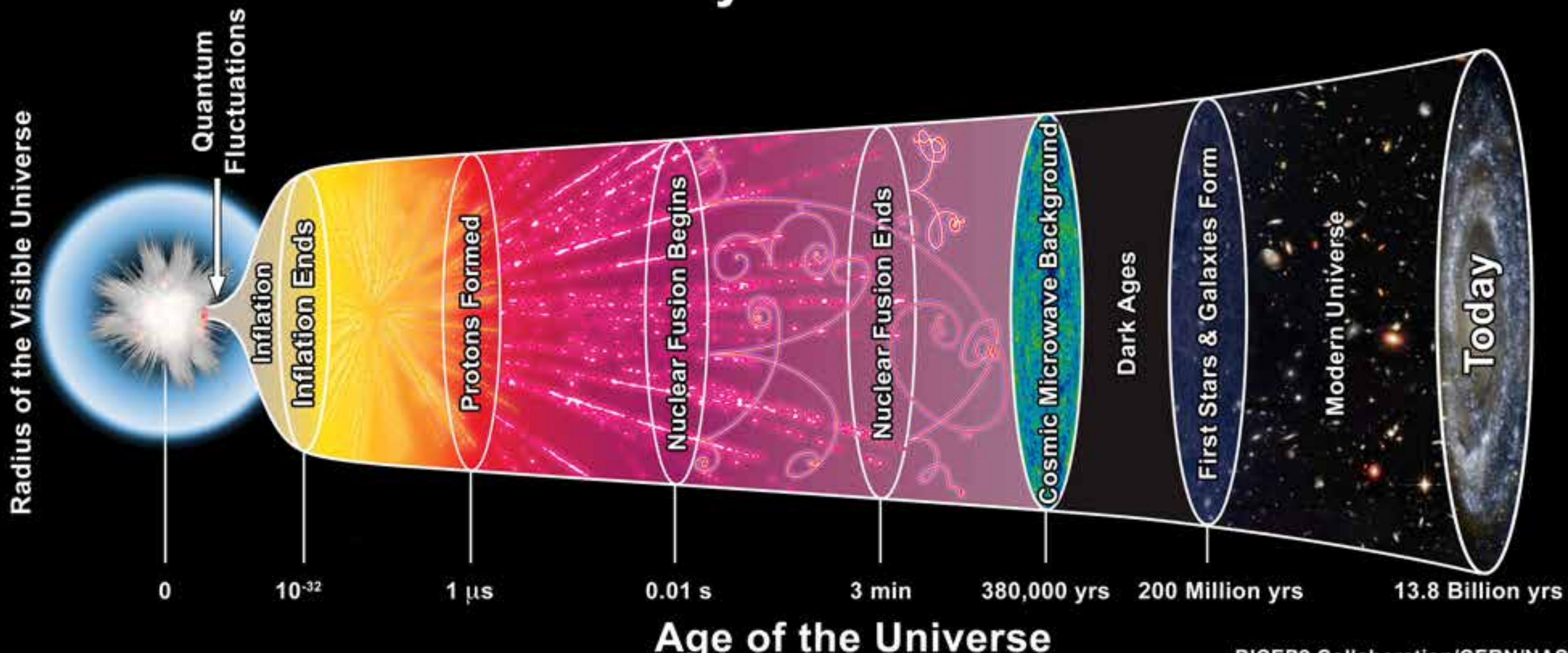
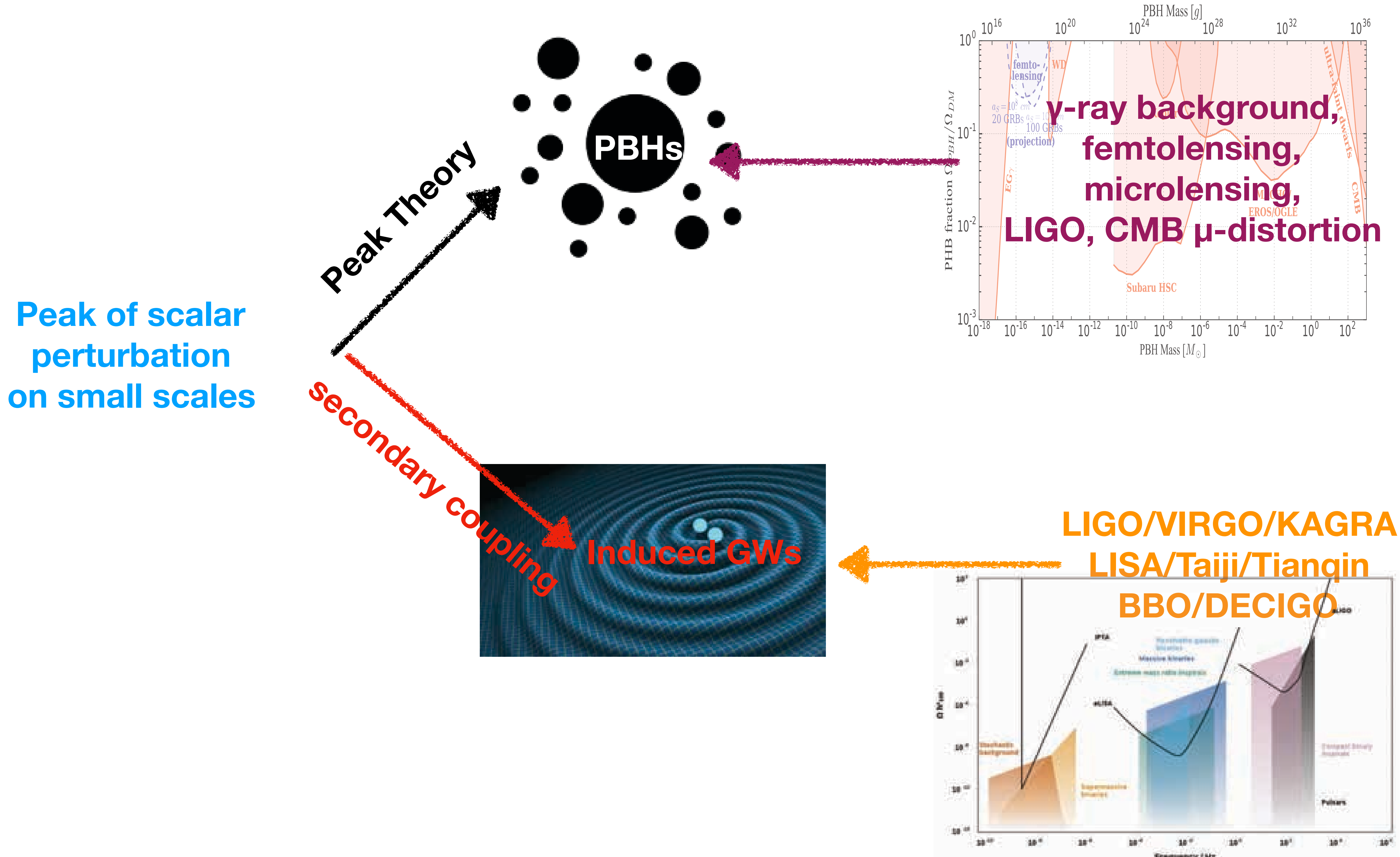
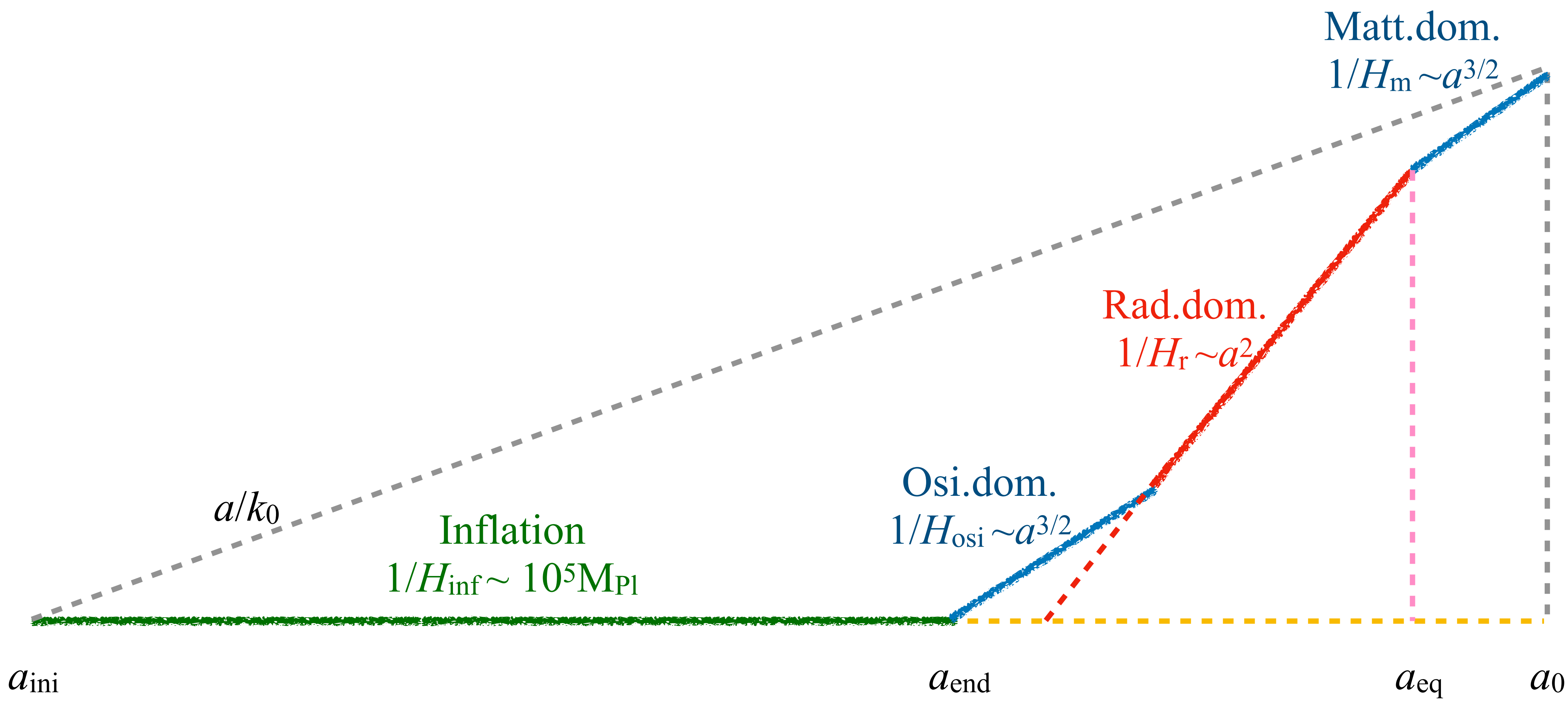
History of the Universe



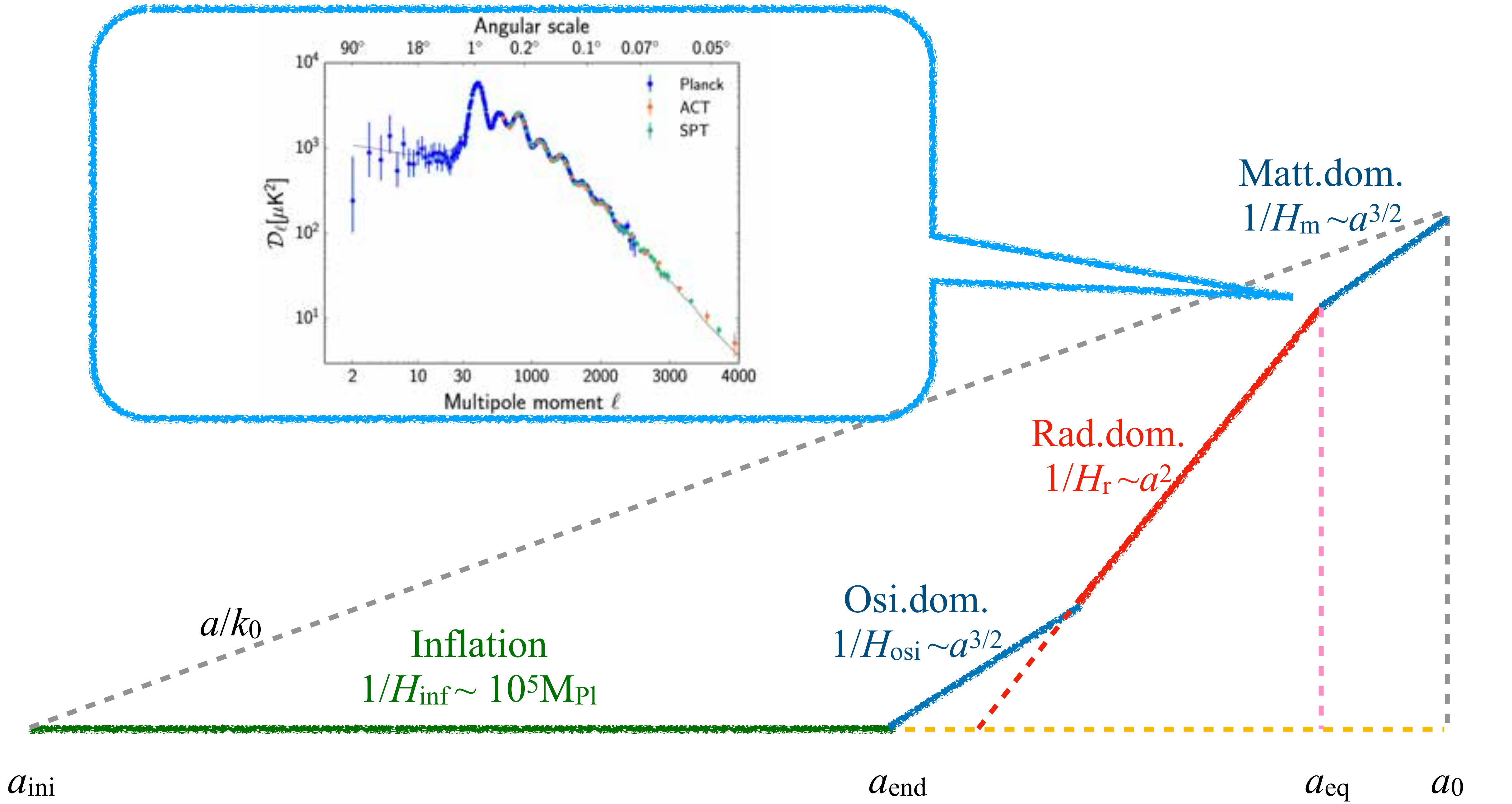
BICEP2 Collaboration/CERN/NASA

Primordial Black Holes

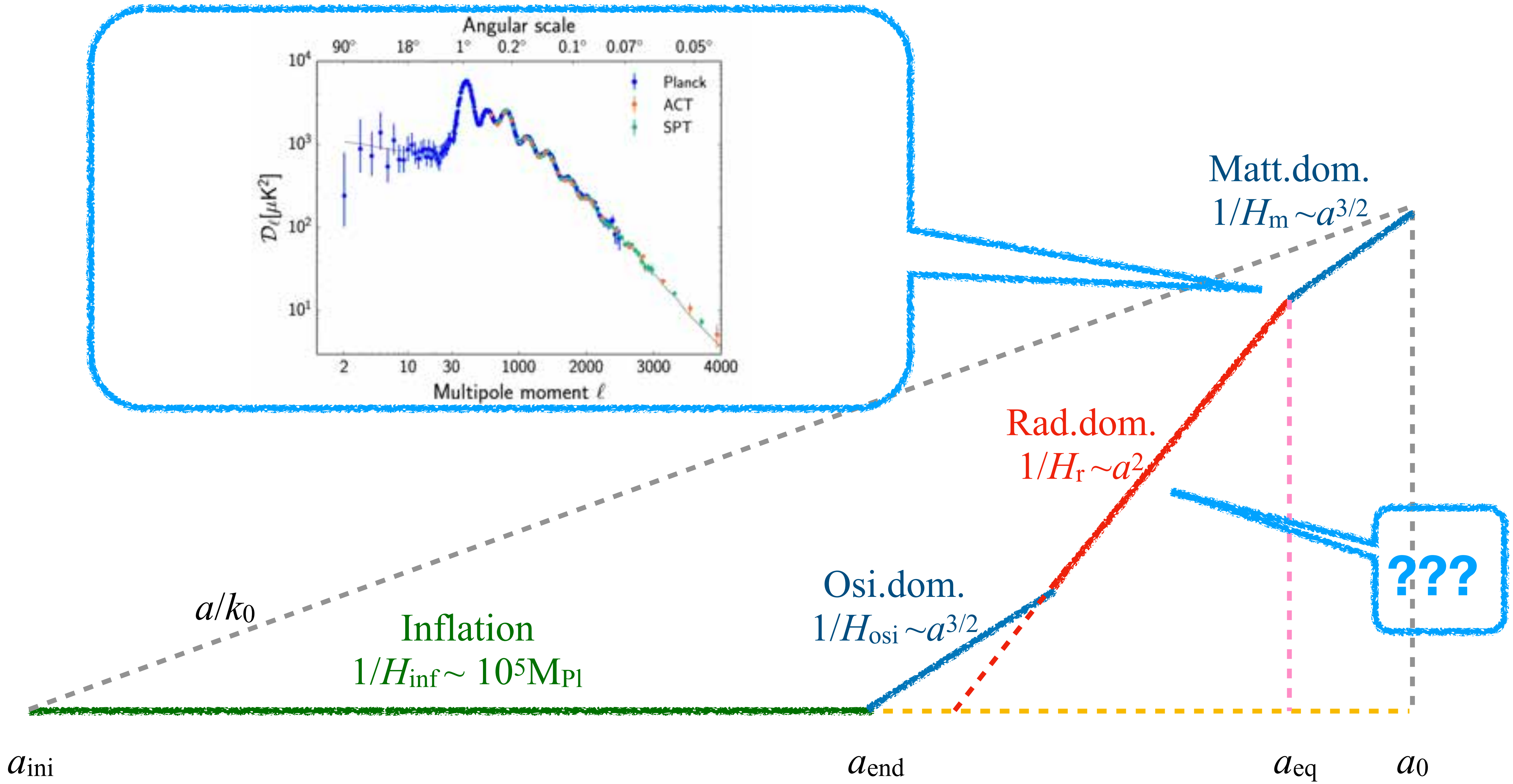




Matt.dom.
$1/H_{\rm m} \sim a^{3/2}$
Rad.dom.
$1/H_{\rm r} \sim a^2$
Osi.dom.
$1/H_{\rm osi} \sim a^{3/2}$
a/k_0
Inflation
$1/H_{\rm inf} \sim 10^5 {\rm M_{Pl}}$
$a_{\rm ini}$
$a_{\rm end}$
$a_{\rm eq}$
a_0



Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell[\mu K^2]$
Multipole moment ℓ
Matt.dom.
$1/H_{\rm m} \sim a^{3/2}$
Rad.dom.
$1/H_{\rm r} \sim a^2$
Osi.dom.
$1/H_{\rm osi} \sim a^{3/2}$
a/k_0
Inflation
$1/H_{\rm inf} \sim 10^5 M_{\rm Pl}$
$a_{\rm ini}$
$a_{\rm end}$
$a_{\rm eq}$
a_0



Angular scale
90°
18°
1°
0.2°
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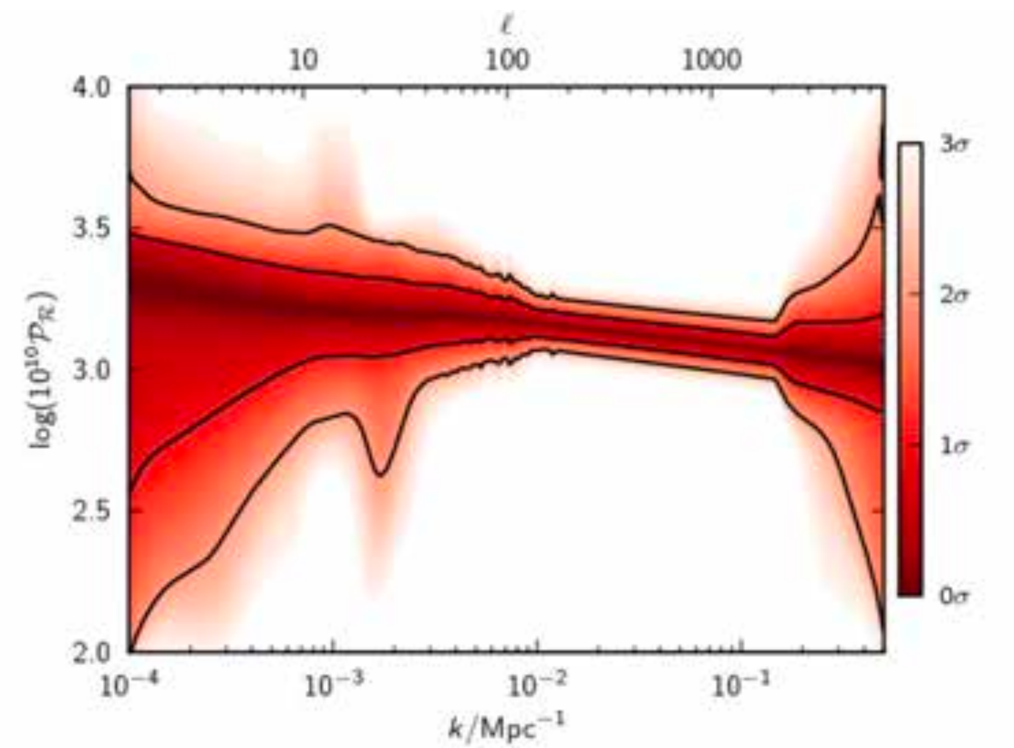
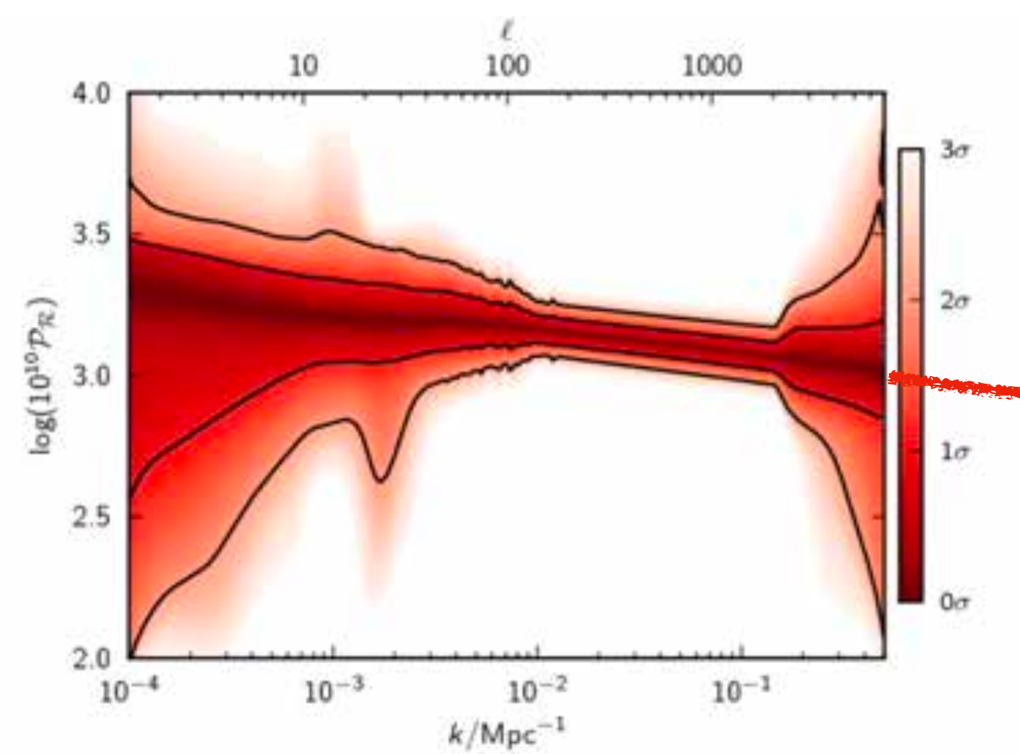
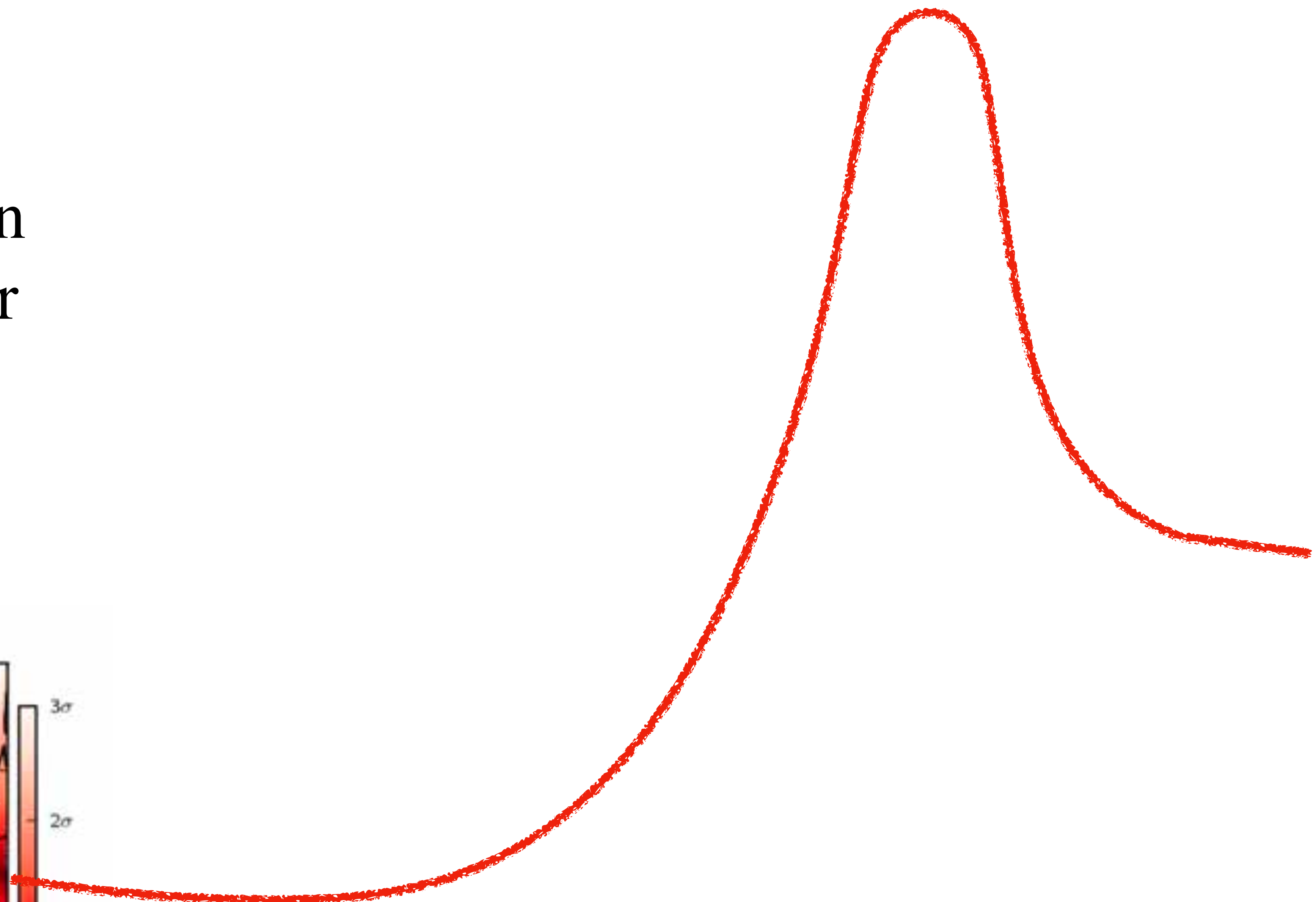
0.07°
0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell [\mu K^2]$
Multipole moment ℓ
Matt.dom.
$1/H_m \sim a^{3/2}$
Rad.dom.
$1/H_r \sim a^2$
???
Osi.dom.
$1/H_{osi} \sim a^{3/2}$
a/k_0
Inflation
$1/H_{inf} \sim 10^5 M_{Pl}$
a_{ini}
a_{end}
a_{eq}
a_0

Bayesian reconstruction of the primordial power spectrum for l<2300. (Planck 2015)



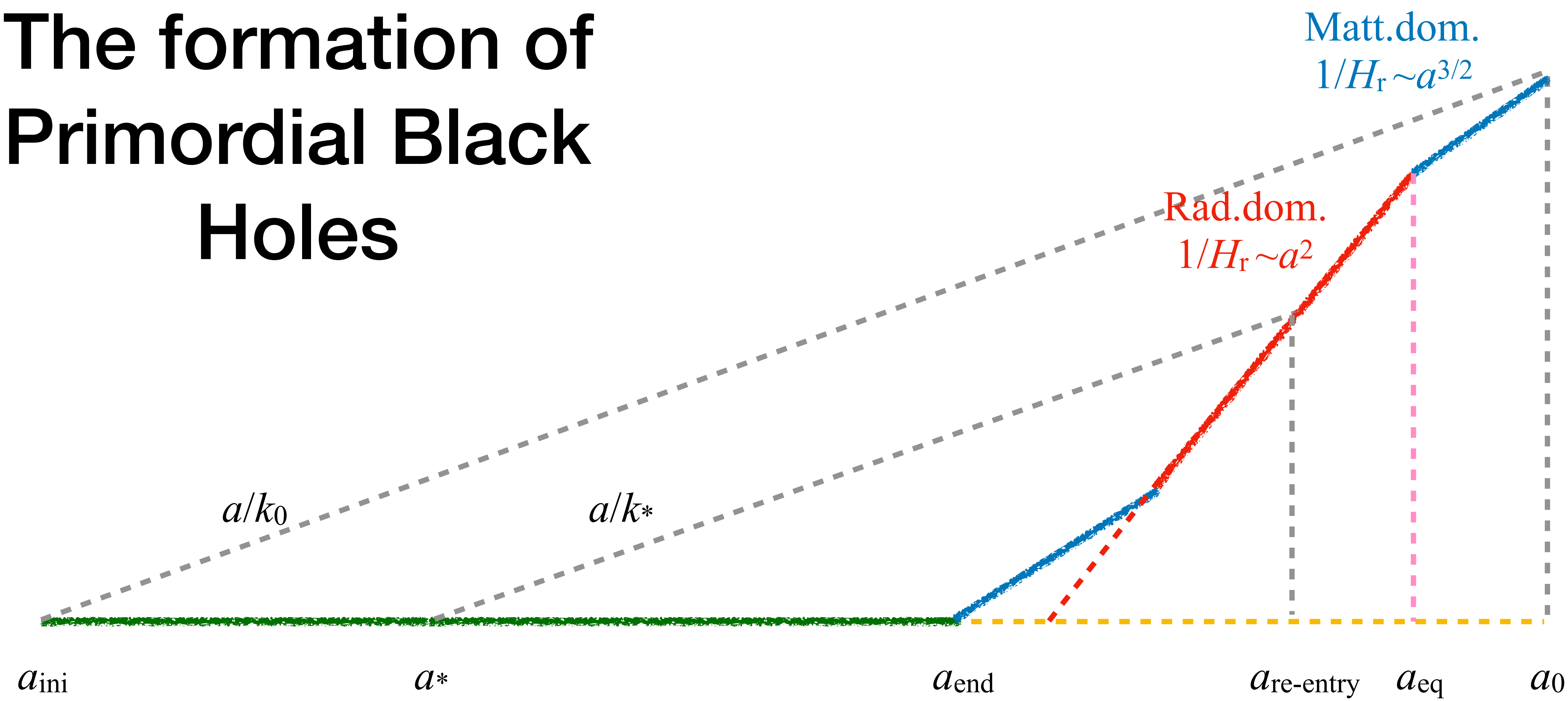
Bayesian reconstruction of the primordial power spectrum for l<2300. (Planck 2015)



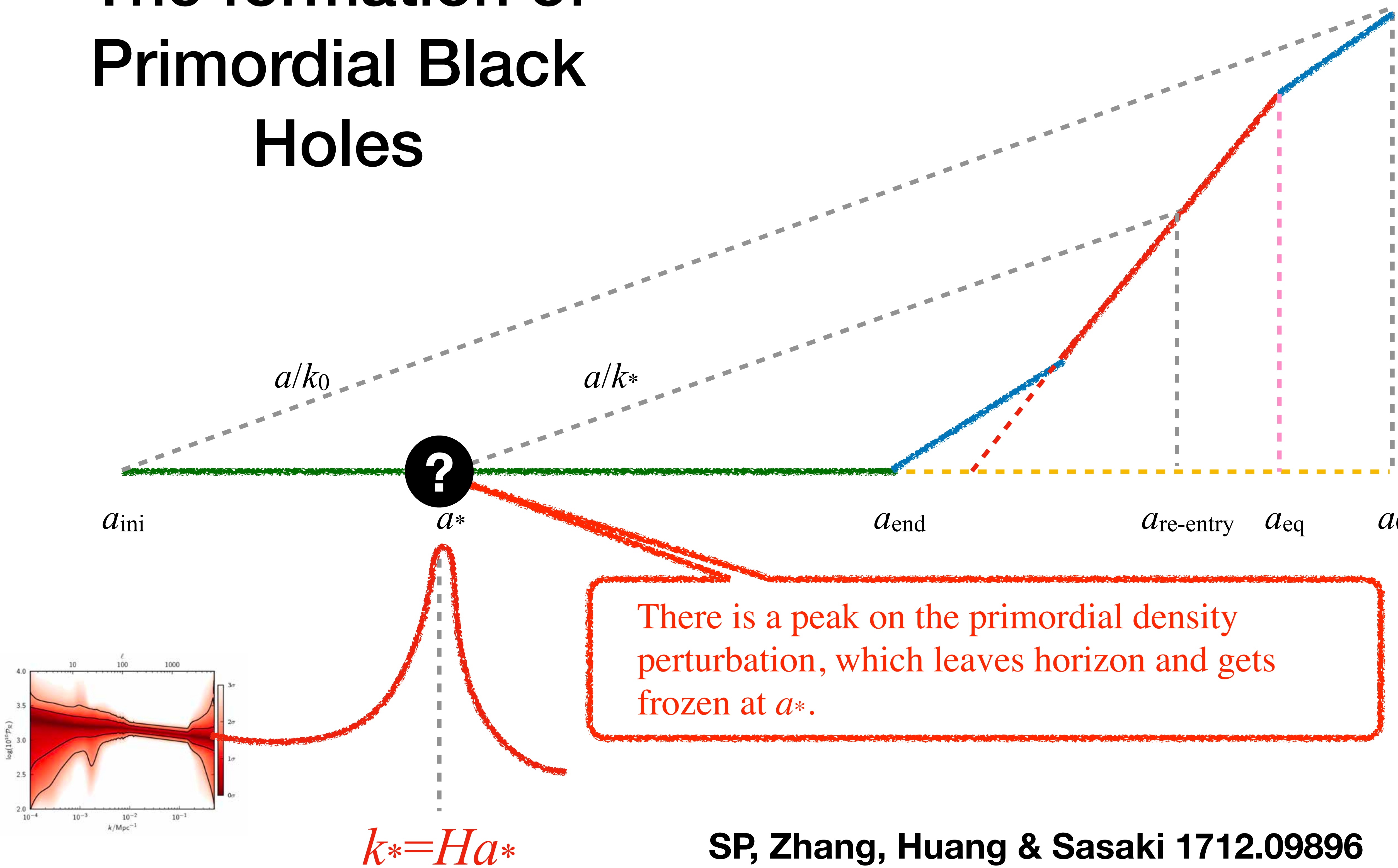


The resolution is lacking to say anything precise about higher l.

The formation of Primordial Black Holes

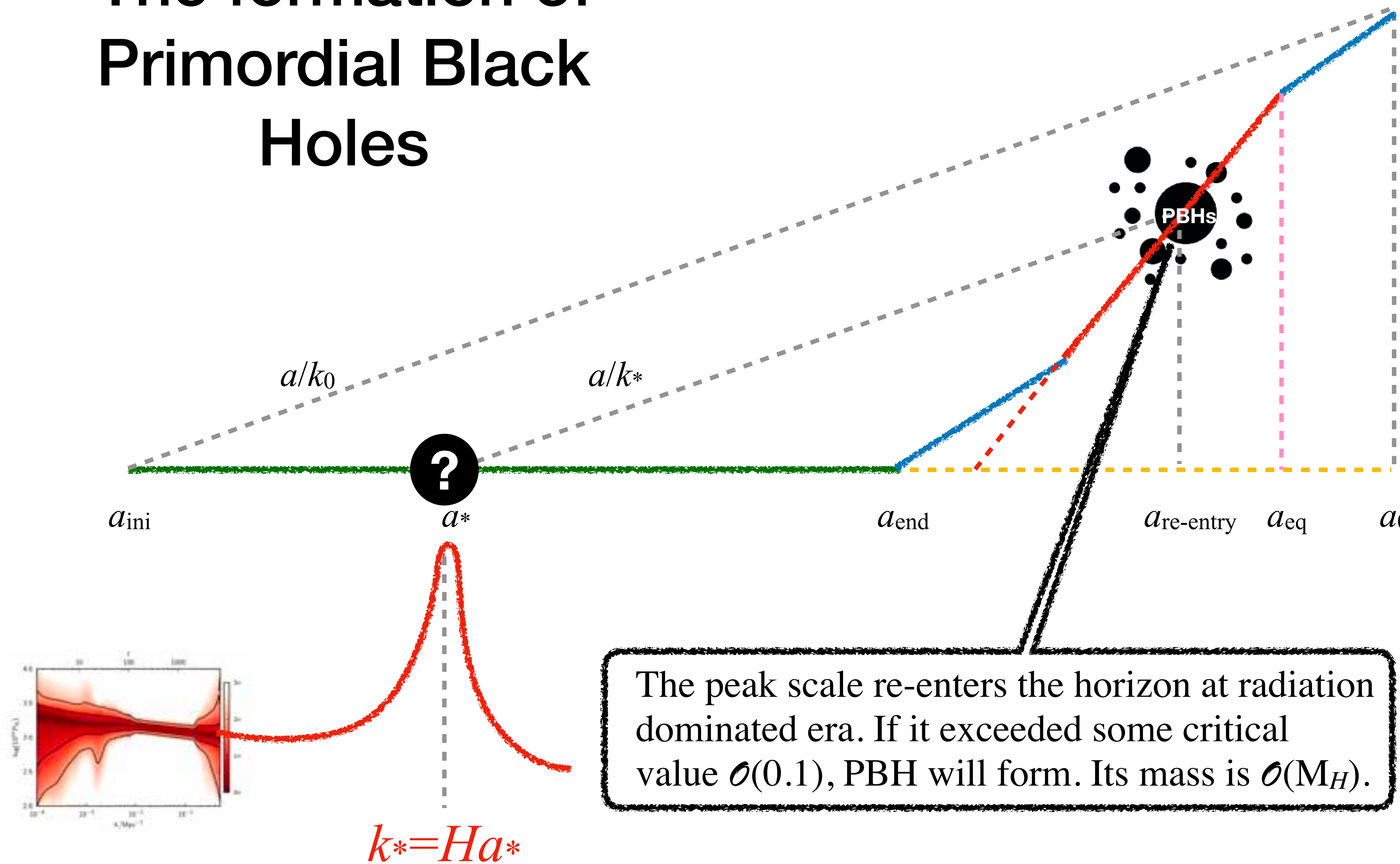


The formation of Primordial Black Holes

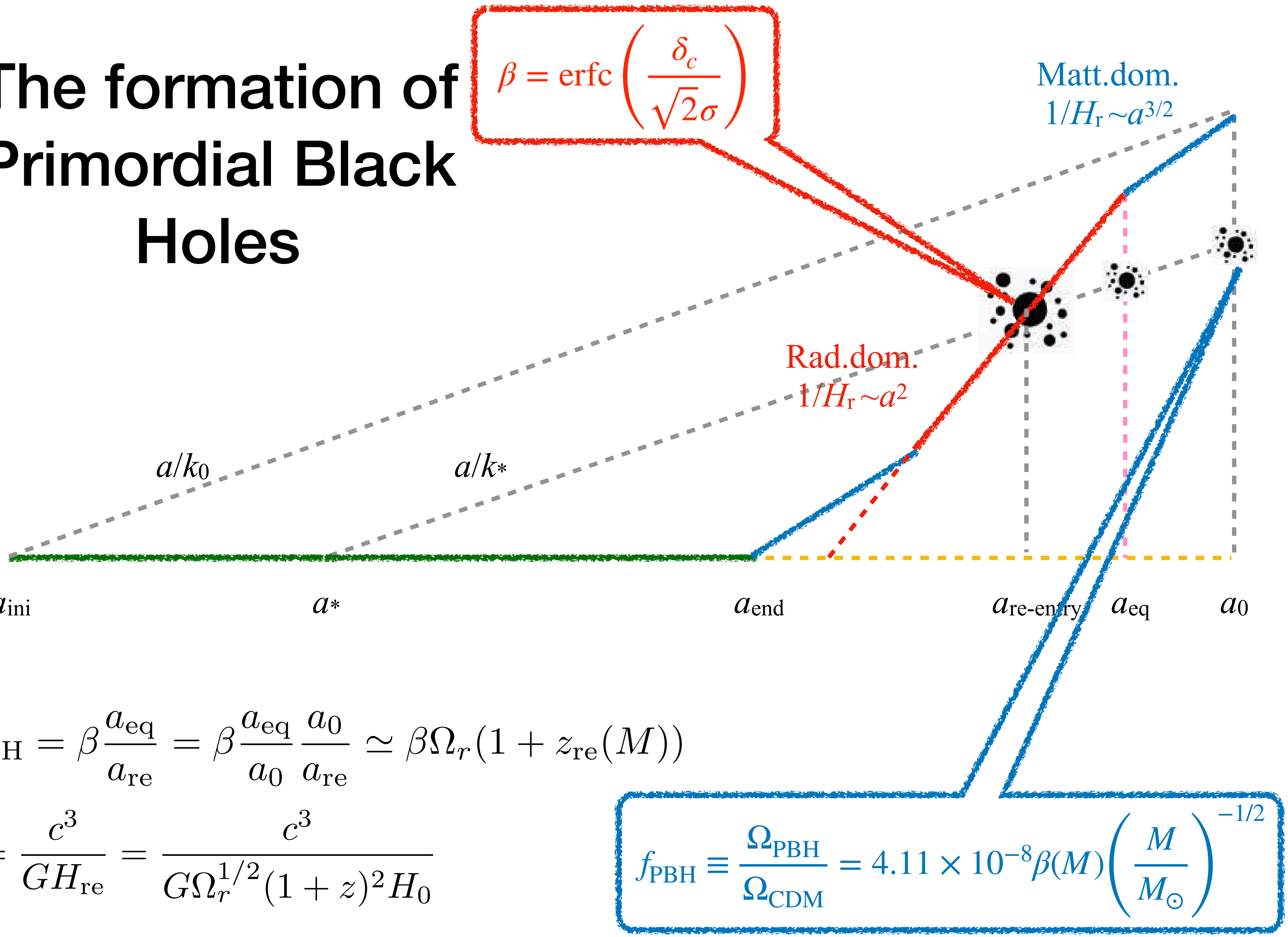


SP, Zhang, Huang & Sasaki 1712.09896

The formation of Primordial Black Holes



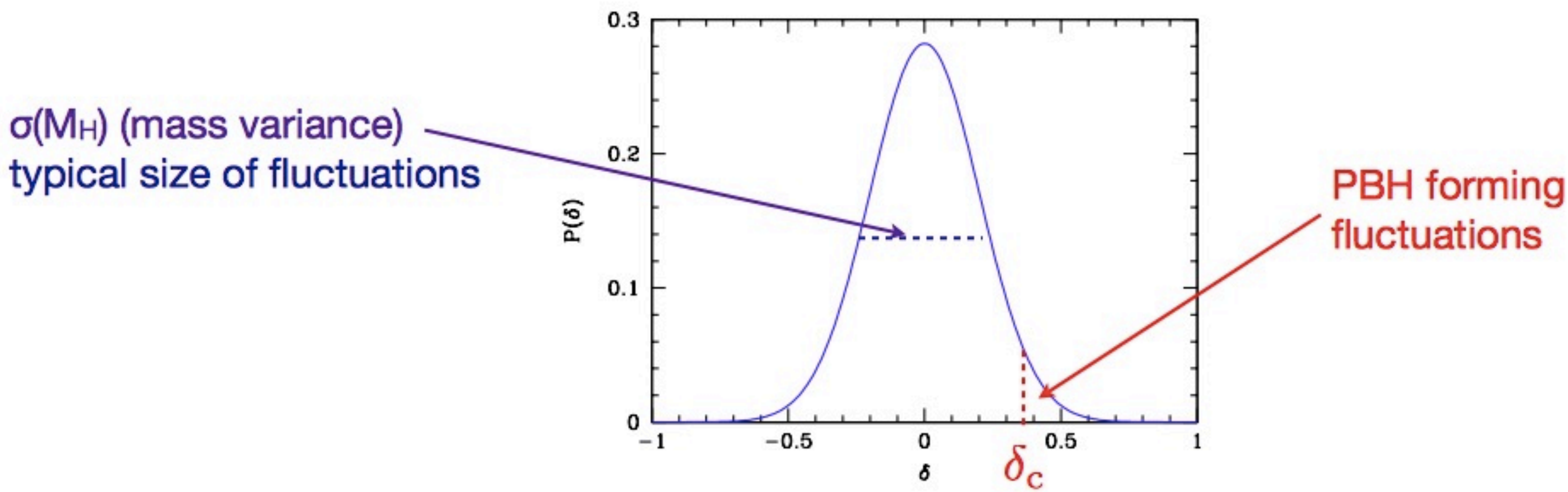
The formation of Primordial Black Holes



$$\Omega_{PBH} = \beta \frac{a_{eq}}{a_{re}} = \beta \frac{a_{eq}}{a_0} \frac{a_0}{a_{re}} \simeq \beta \Omega_r (1 + z_{re}(M))$$

$$M = \frac{c^3}{G H_{re}} = \frac{c^3}{G \Omega_r^{1/2} (1+z)^2 H_0}$$

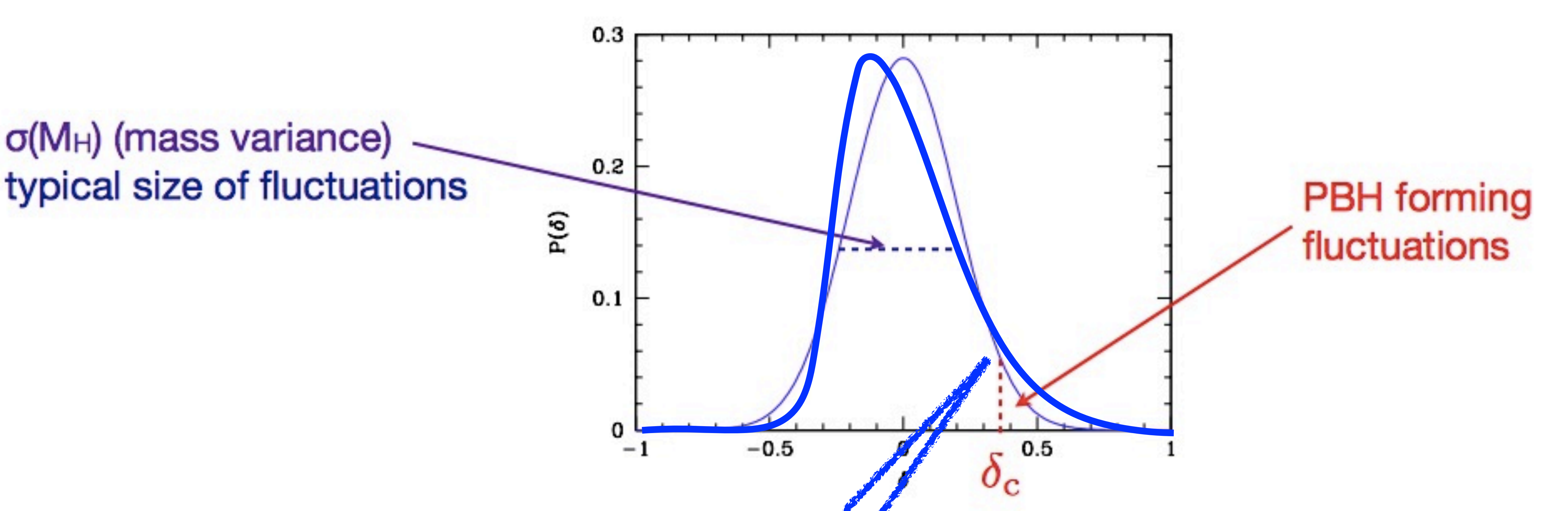
The Press-Schechter Mass Function



- When $\sigma_M << \delta_c$, β can be approximated by exponential:

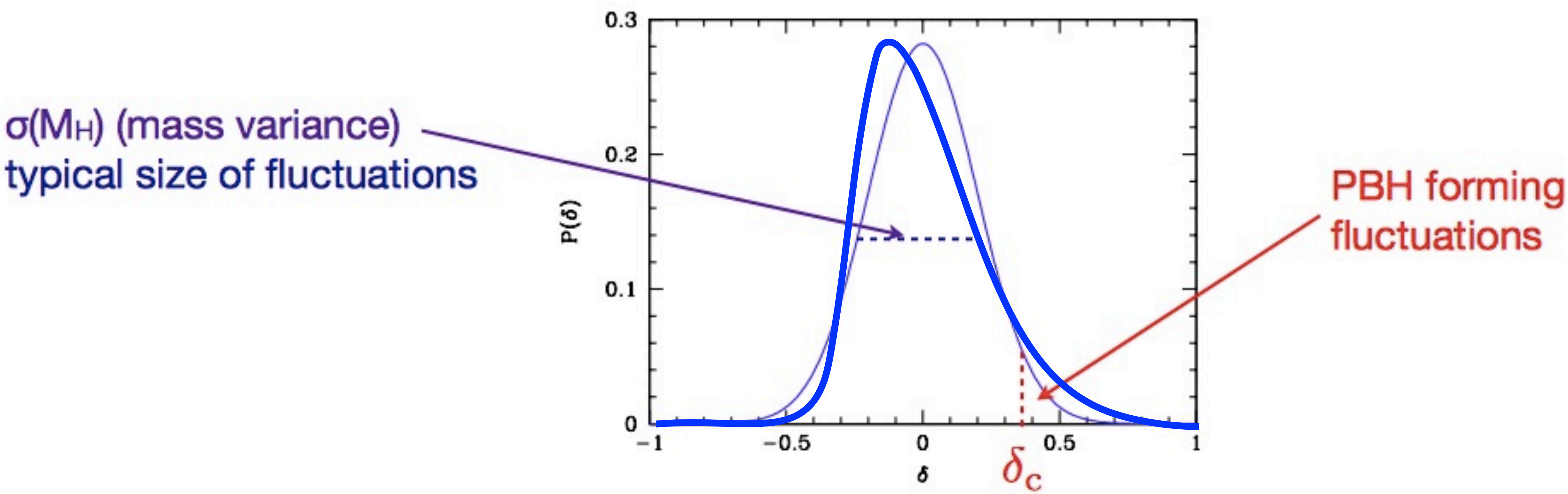
$$\beta \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

The Press-Schechter Mass Function



Non-Gaussianity can increase (f_{NL}>0) or decrease (f_{NL}<0) the PBH adundances.

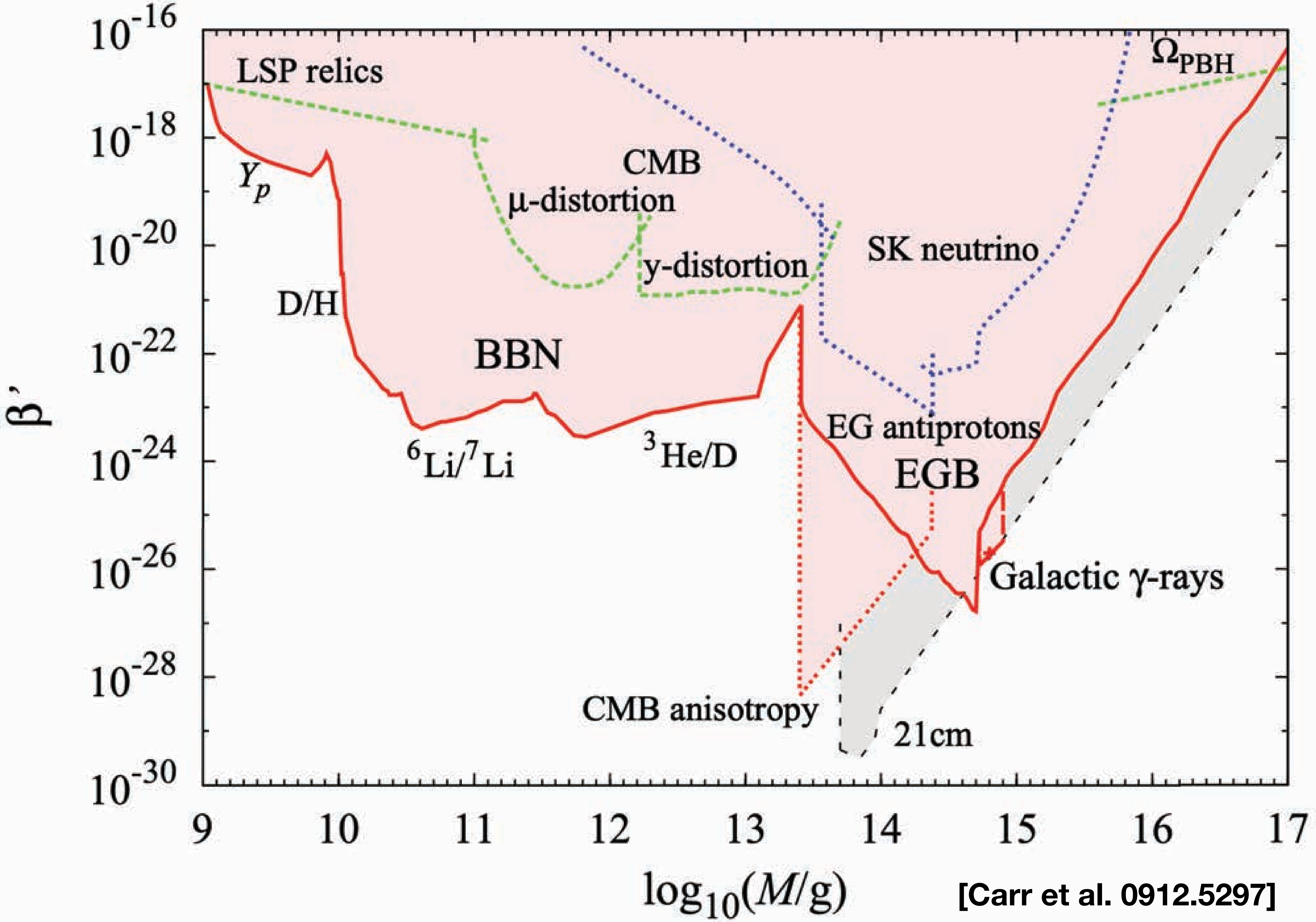
The Press-Schechter Mass Function



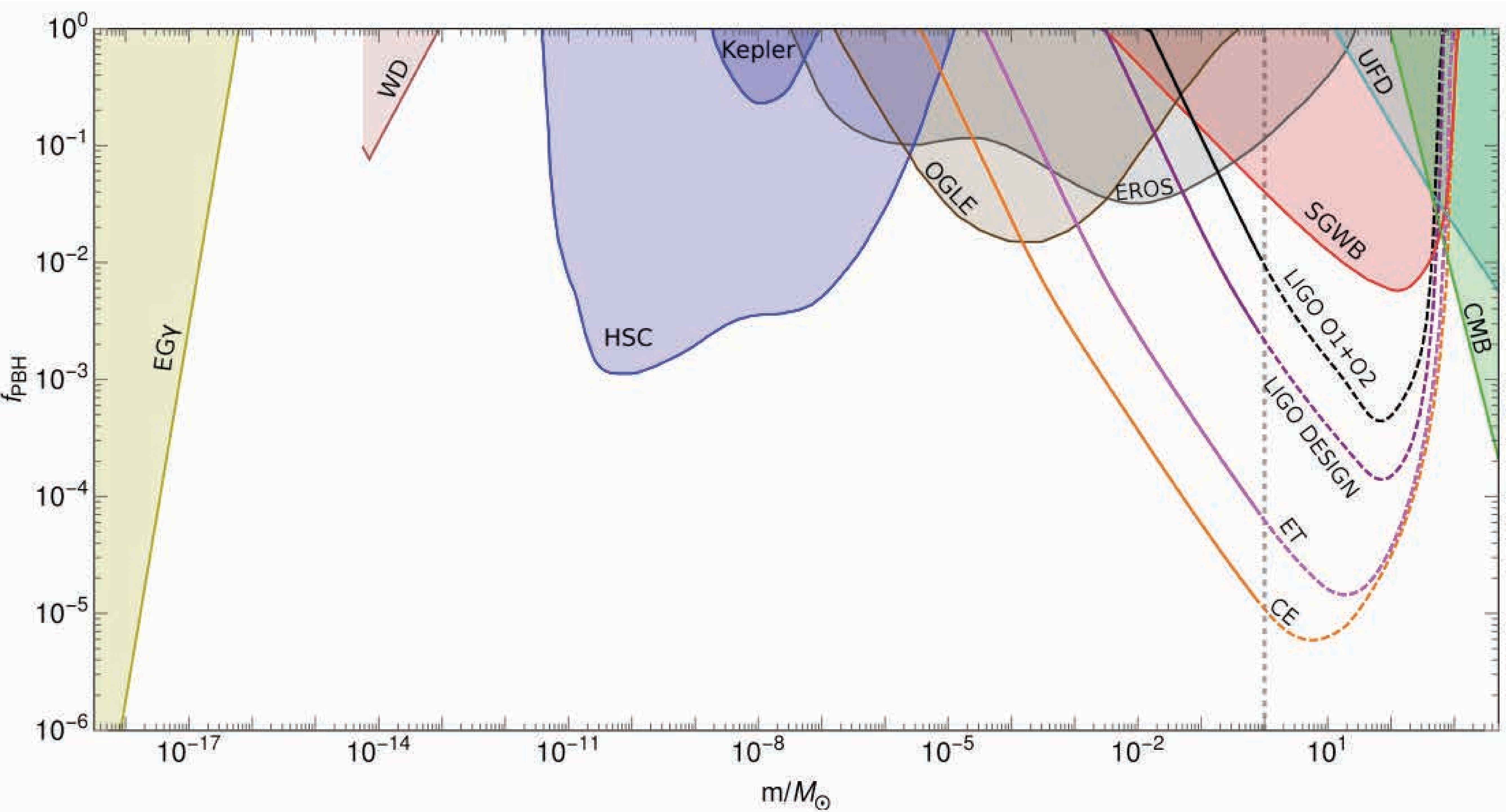
$$\mathscr{R}_{g\pm}(\mathscr{R}) = \frac{1}{2} f_{\rm NL}^{-1} \left(-1 \pm \sqrt{1 + 4 f_{\rm NL} \left(f_{\rm NL} \mathscr{A}_{\mathscr{R}} + \mathscr{R} \right)} \right).$$

$$\beta = \frac{1}{2} \mathrm{erfc} \left(\frac{\mathscr{R}_{g+}(\mathscr{R}_c)}{\sqrt{2\mathscr{A}_{\mathscr{R}}}} \right) - \frac{1}{2} \mathrm{erfc} \left(-\frac{\mathscr{R}_{g-}(\mathscr{R}_c)}{\sqrt{2\mathscr{A}_{\mathscr{R}}}} \right); \quad f_{\rm NL} > 0.$$

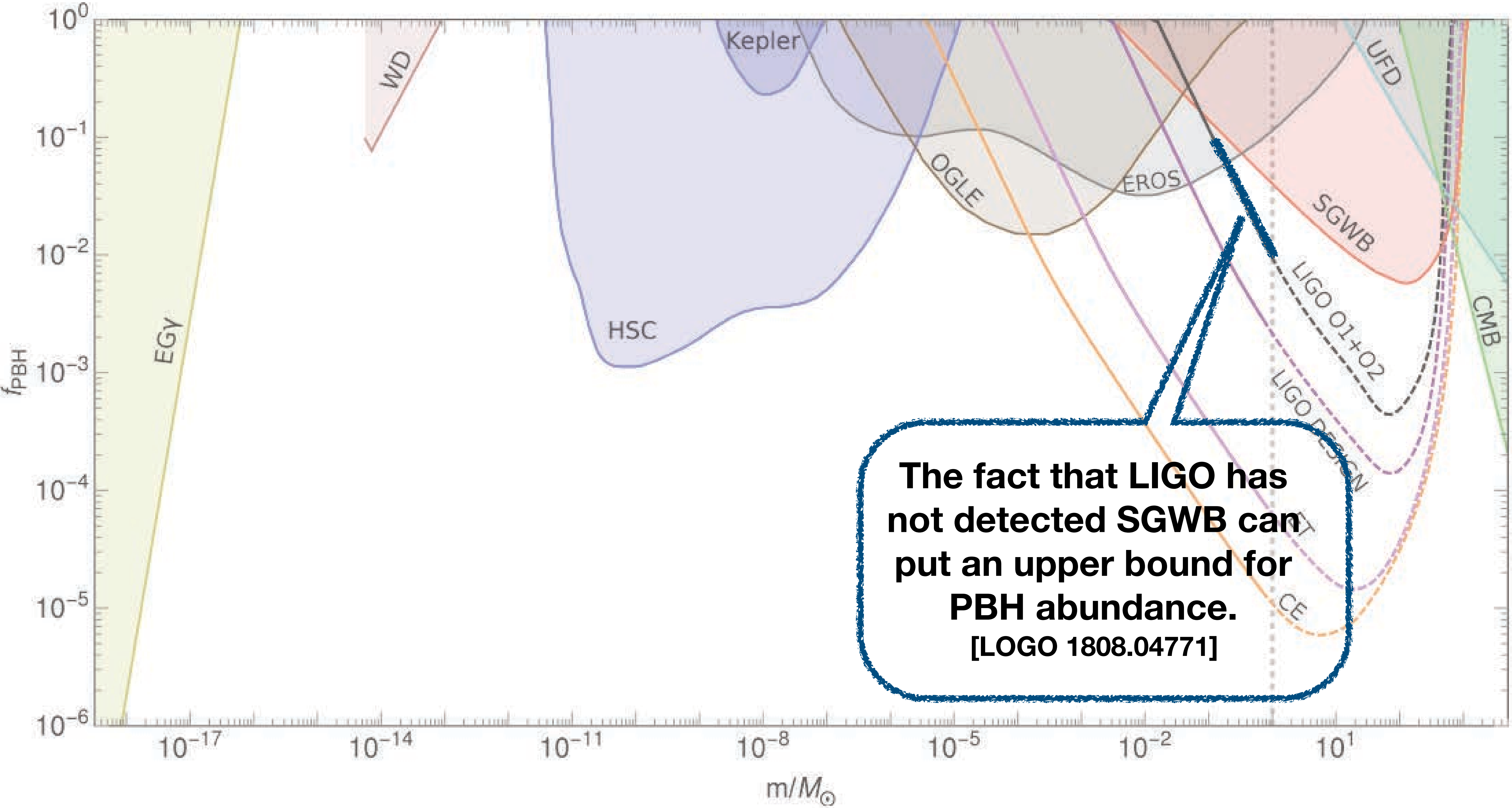
[Young & Byrnes, 1307.4995]



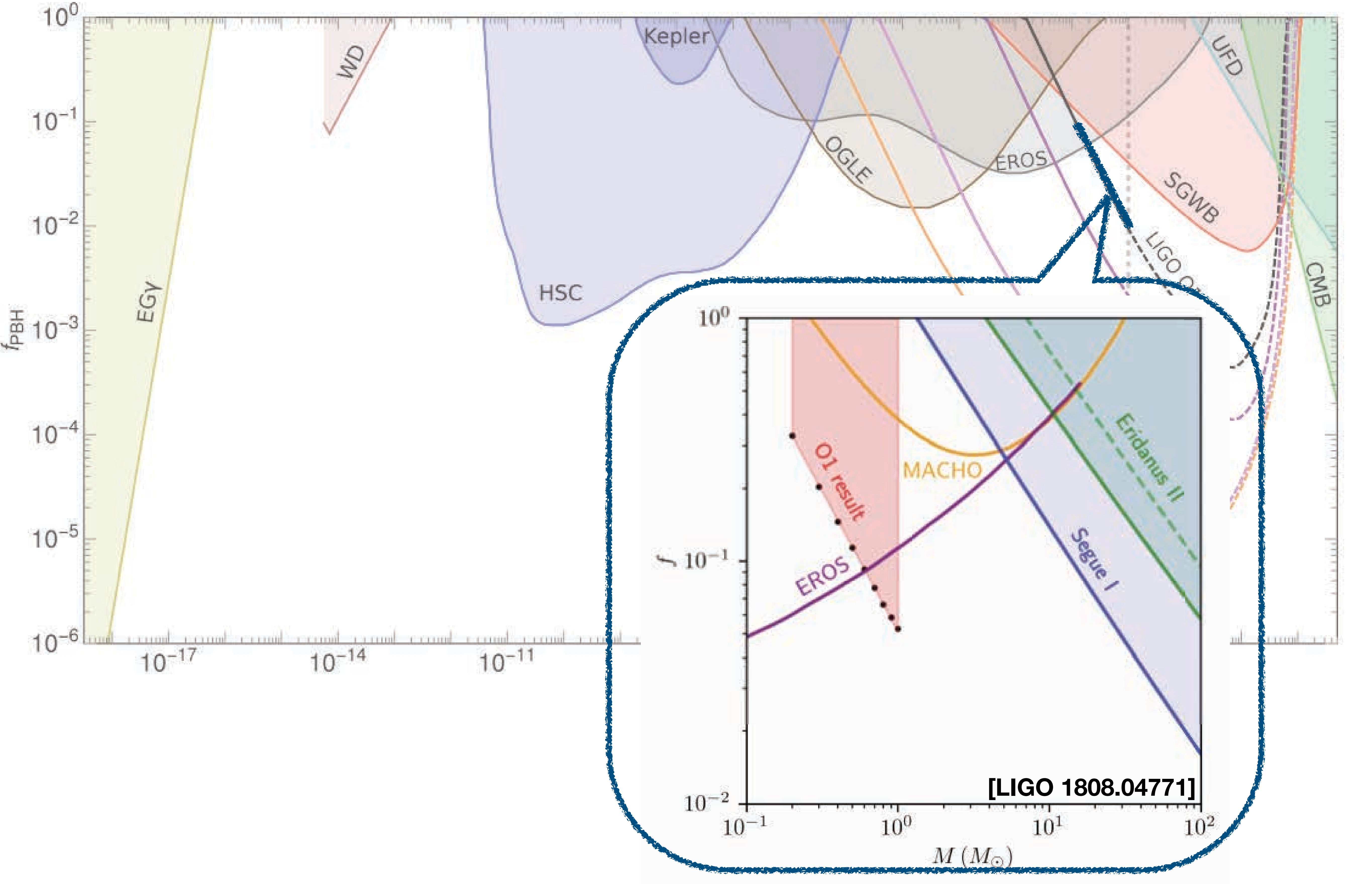
[Carr et al. 0912.5297]



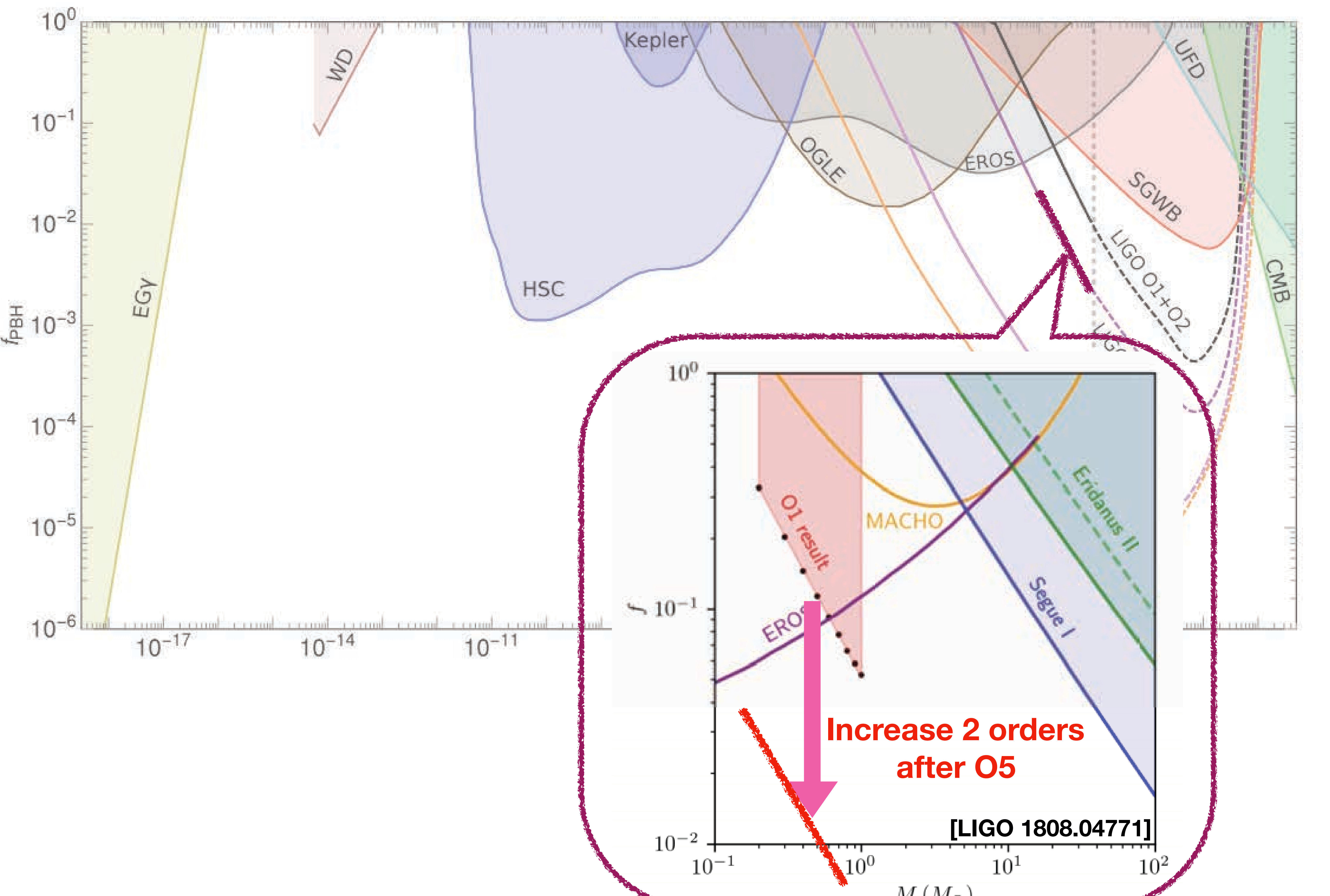
[Chen and Huang. 1904.02396]



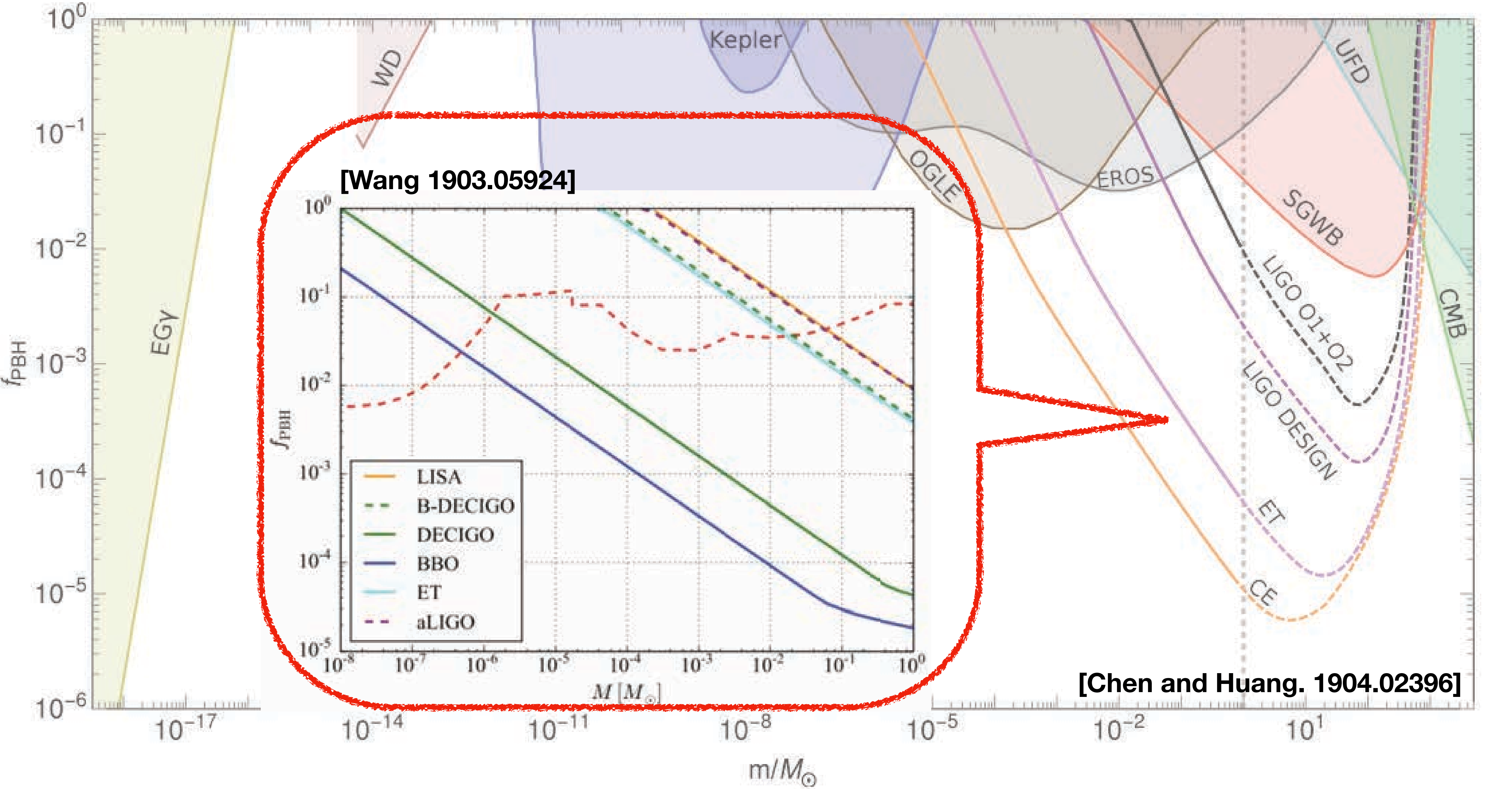
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
f_{PBH}
10^{-17}
10^{-14}
10^{-11}
10^{-8}
10^{-5}
10^{-2}
10^{1}
$m/M_{\odot}$
EGγ
WD
HSC
Kepler
OGLE
EROS
SGWB
UFD
CMB
LIGO O1+O2
CE
The fact that LIGO has not detected SGWB can put an upper bound for PBH abundance.
[LOGO 1808.04771]



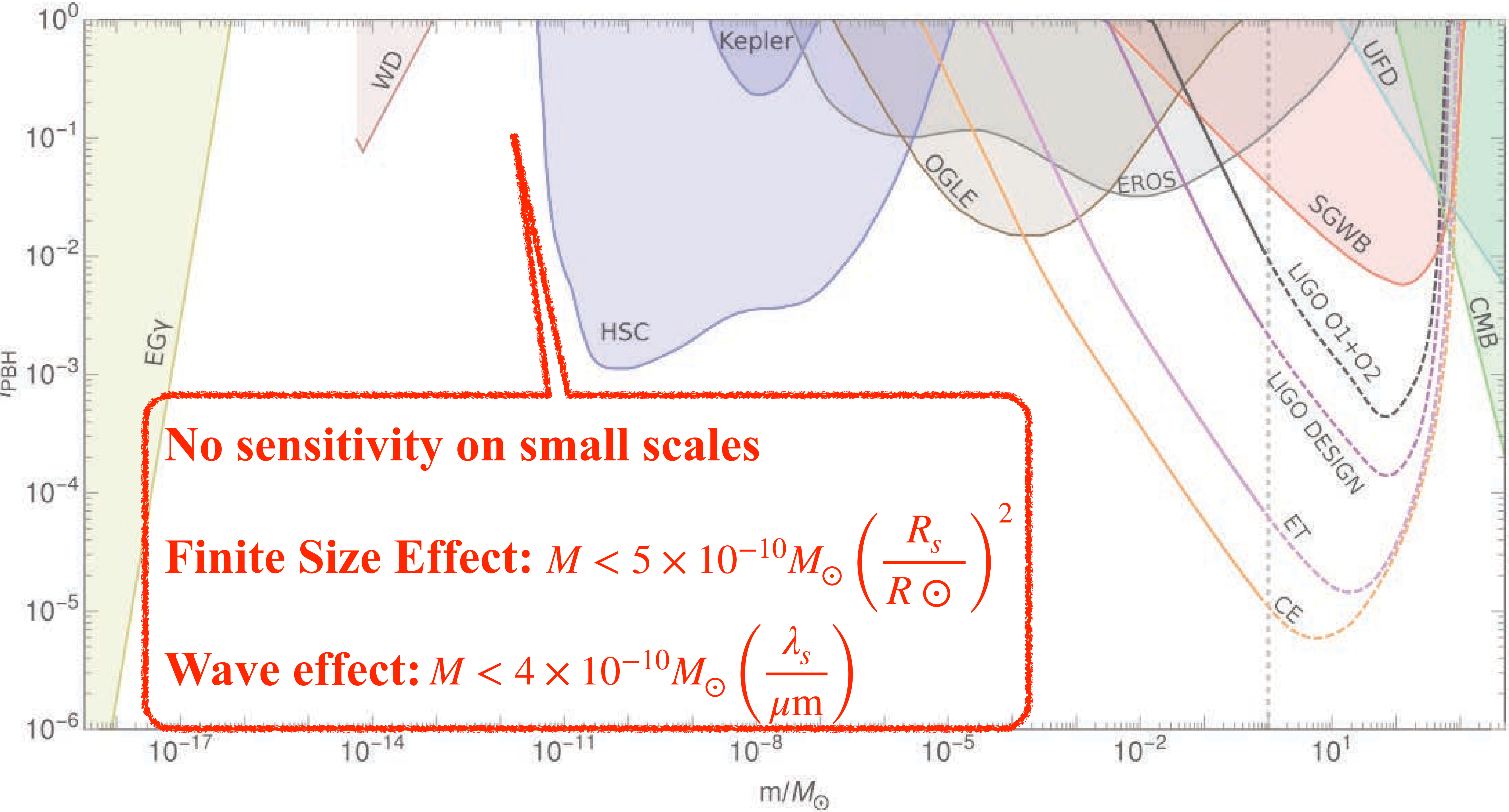
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
f_{PBH}
10^{-17}
10^{-14}
10^{-11}
EGγ
WD
Kepler
HSC
OGLE
EROS
SGWB
UFD
CMB
LIGO O
10^0
10^{-1}
10^{-2}
f
10^{-1}
10^0
10^1
10^2
$M\,(M_\odot)$
O1 result
MACHO
EROS
Eridanus II
Segue I
[LIGO 1808.04771]



EGγ
WD
HSC
Kepler
OGLE
EROS
UFD
SGWB
LIGO O1+O2
CMB
$f_{\rm PBH}$
MACHO
O1 result
Eridanus II
Segue I
EROS
Increase 2 orders after O5
[LIGO 1808.04771]



[Wang 1903.05924]
LISA
B-DECIGO
DECIGO
BBO
ET
aLIGO
f_{PBH}
$M\ [M_\odot]$
Kepler
WD
OGLE
EROS
SGWB
UFD
CMB
EGγ
LIGO O1+O2
LIGO DESIGN
ET
CE
$m/M_\odot$
[Chen and Huang. 1904.02396]

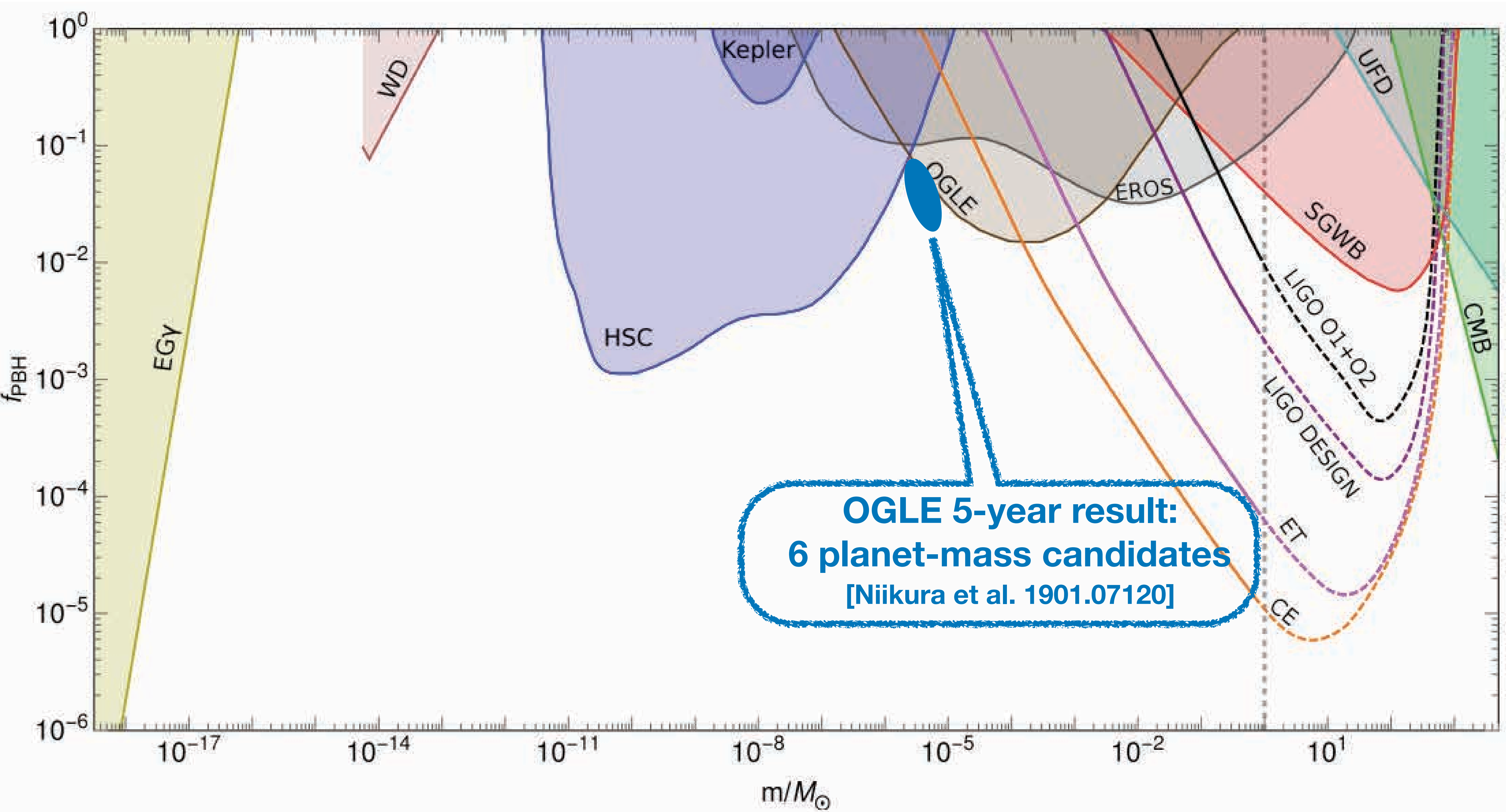


No sensitivity on small scales

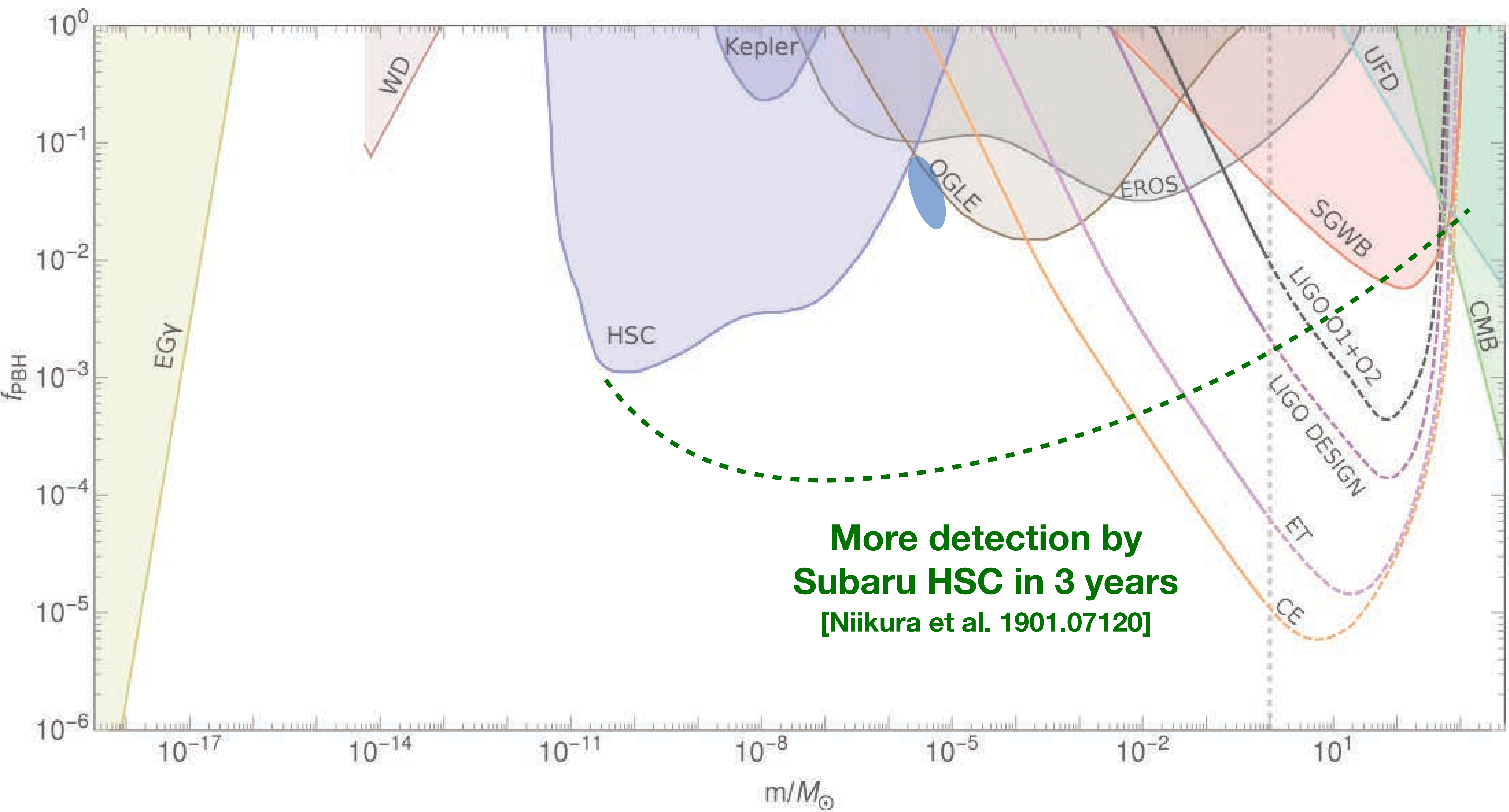
Finite Size Effect: $M < 5 \times 10^{-10} M_\odot \left(\frac{R_s}{R\odot}\right)^2$

Wave effect: $M < 4 \times 10^{-10} M_\odot \left(\frac{\lambda_s}{\mu \mathrm{m}}\right)$

[Chen and Huang. 1904.02396]



[Chen and Huang. 1904.02396]



[Chen and Huang. 1904.02396]

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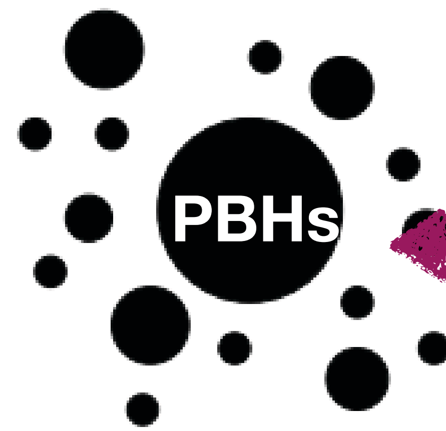
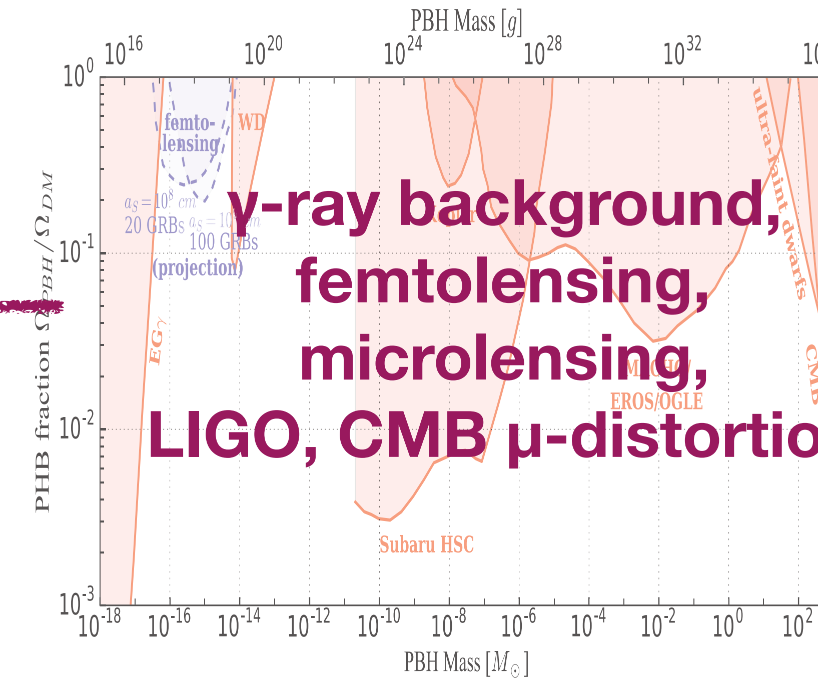
Induced GWs

Peak of scalar perturbation on small scales

Peak Theory

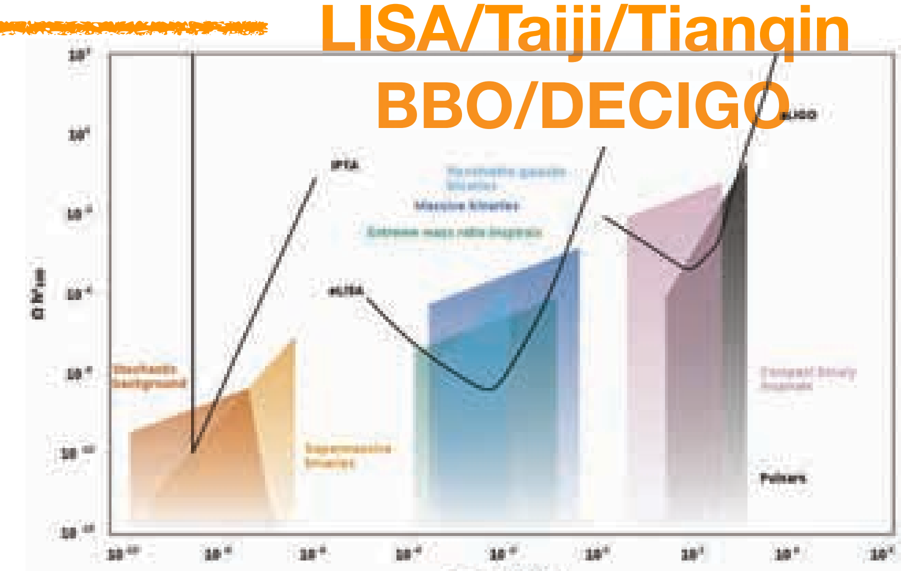


γ-ray background, femtolensing, microlensing, LIGO, CMB μ-distortion

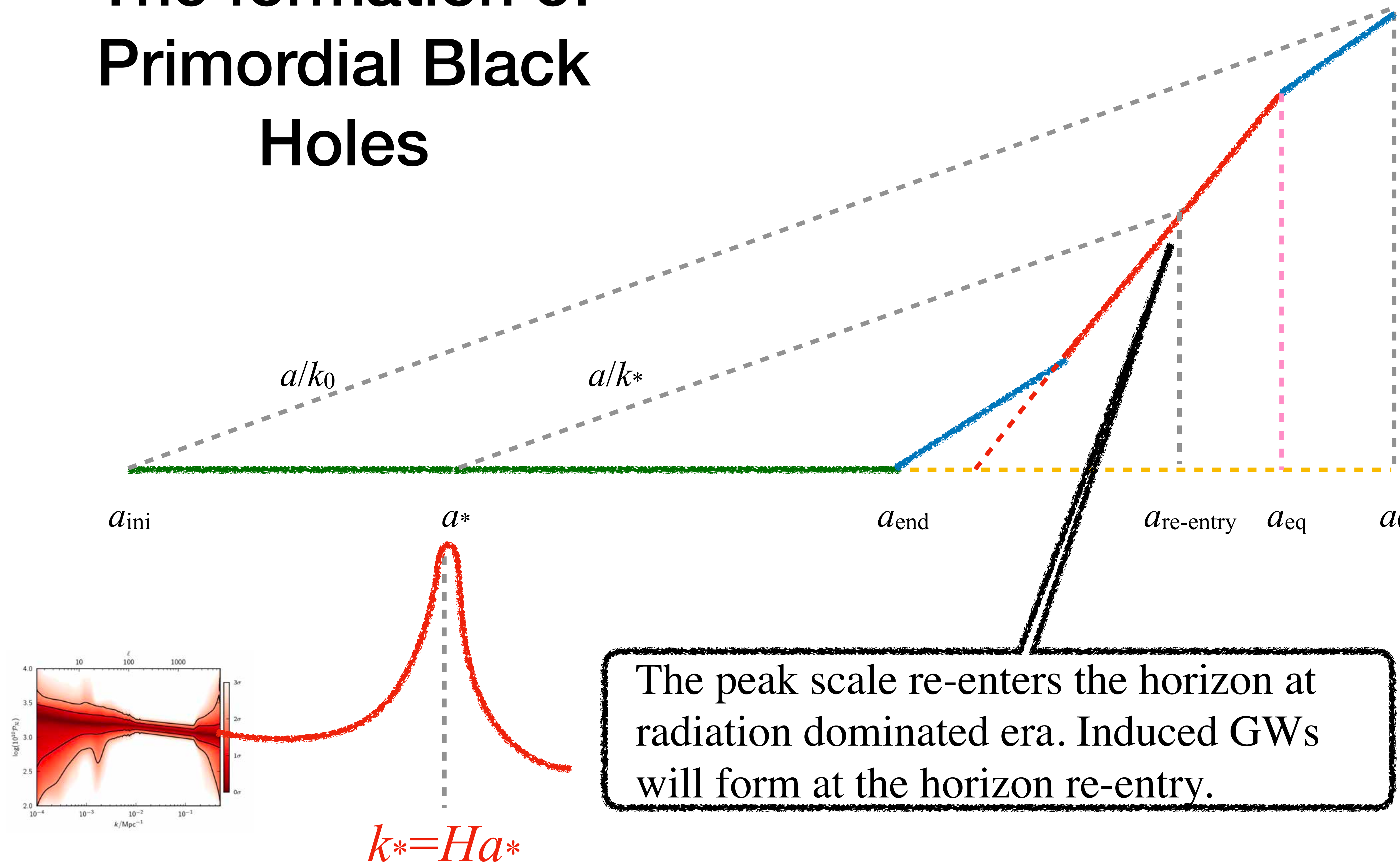


secondary coupling

LIGO/VIRGO/KAGRA
LISA/Taiji/Tianqin
BBO/DECIGO



The formation of Primordial Black Holes



Induced GWs

- The metric is

$$ds^2 = a(\eta)^2 \left[-(1-2\Phi)\, d\eta^2 + \left(1 + 2\Phi + \frac{1}{2} h_{ij}\right) dx^i dx^j \right] .$$

- From the nonlinear equation of motion for the tensor perturbation

$$h''_{\mathbf{k}} + 2\mathscr{H} h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathscr{S}(\mathbf{k}, \eta)$$

- where the source term is

$$\mathcal{S}(\mathbf{k}, \eta) = 36 \int \frac{d^3 l}{(2\pi)^{3/2}} \frac{l^2}{\sqrt{2}} \sin^2\theta \begin{pmatrix} \cos 2\varphi \\ \sin 2\varphi \end{pmatrix} \Phi_{\mathbf{l}} \Phi_{\mathbf{k}-\mathbf{l}}$$
$$\times \left[j_0(ux) j_0(vx) - 2 \frac{j_1(ux) j_0(vx)}{ux} - 2 \frac{j_0(ux) j_1(vx)}{vx} + 3 \frac{j_1(ux) j_1(vx)}{uvx^2} \right] .$$

Induced GWs

- The solution to the eom of $h_{\mathbf{k}}$ is

$$h_{\mathbf{k}} = \frac{(2\pi)^{3/2}}{k\eta}\left(\mathcal{S}'_{\mathbf{k}}(k)e^{ik\eta} - \mathcal{S}'_{\mathbf{k}}(-k)e^{-ik\eta}\right).$$

- Then we know that Ω_{GW}~<hh>~<SS>~<ΦΦΦΦ>~P_{Φ}^2:

$$\Omega_{\rm GW} = \frac{k^3}{2}\left(\frac{H_{\rm eq}}{H_0}\right)^2\left(\frac{a_{\rm eq}}{a_0}\right)^4 \Re\iint d\eta d\tau\ \eta\tau e^{-ik\eta+ip\tau}\left\langle \mathcal{S}_{\mathbf{k}}(\eta)\mathcal{S}^*_{\mathbf{p}}(\tau)\right\rangle'.$$

$$\sim k^3\int d\eta\int d\tau \times \text{Green function} \times \mathscr{P}^2_{\Phi}.$$

Induced GWs

- The solution to the eom of $h_{\mathbf{k}}$ is

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$$\sim k^3 \int d\eta \int d\tau \times \text{Green function} \times \mathcal{P}_{\Phi}^2.$$

- Why Gaussian?

Induced GWs

- Therefore we want to consider the local-type non-Gaussian scalar induced GWs.

$$\mathscr{R}(\mathbf{x}) = \mathscr{R}_g(\mathbf{x}) + F_{\rm NL}\left[\mathscr{R}_g^2(\mathbf{x}) - \langle \mathscr{R}_g^2(\mathbf{x})\rangle\right].$$

$$\langle \Phi_{\mathbf{k}}\Phi_{\mathbf{p}}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{p})\frac{4}{9}\left(P_{\mathscr{R}}(k) + 2F_{\rm NL}^2\int d^3l\; P_{\mathscr{R}}(|\mathbf{k}-\mathbf{l}|)P_{\mathscr{R}}(l)\right).$$

- And we have to specify the power spectrum of the primordial curvature perturbation. As we mentioned, we suppose there is a narrow peak at around k*.

$$P_{\mathscr{R}}(k) = \frac{\mathscr{A}_{\mathscr{R}}}{(2\pi)^{3/2}2\sigma k_*^2}\exp\left(-\frac{(k-k_*)^2}{2\sigma^2}\right).$$

Induced GWs

- The convolution of the two power spectra is

$$\mathcal{F} \equiv \frac{\mathcal{A}_{\mathcal{R}}^2}{8\pi k k_*^2} \left\{ \left[\frac{1}{2}\mathrm{erf}\left(\frac{k}{2\sigma}\right) + \frac{\sigma k}{k_*^2}\frac{e^{-\frac{k^2}{4\sigma^2}}}{4\sqrt{\pi}} \right] \mathrm{erfc}\left(-\frac{k_*}{\sigma} + \frac{k}{2\sigma}\right) + \frac{\sigma}{4\sqrt{\pi}k_*}\left(2 + \frac{k}{k_*}\right) e^{\frac{k_*(k-k_*)}{\sigma^2} - \frac{k^2}{4\sigma^2}} \mathrm{erf}\left(\frac{k^2}{2\sigma^2}\right) \right\}$$

- Then one half of the integral is

$$P_{\mathcal{R}} + 2F_{\mathrm{NL}}^2 \int d^3 l P_{\mathcal{R}}(|\mathbf{k} - \mathbf{l}|) P_{\mathcal{R}}(l) = \frac{\mathcal{A}_{\mathcal{R}}}{(2\pi)^{3/2} 2\sigma k_*^2} \left(e^{-\frac{(k-k_*)^2}{2\sigma^2}} + \sqrt{2\pi} F_{\mathrm{NL}}^2 \mathcal{A}_{\mathcal{R}} \frac{\sigma}{k} \mathcal{F}(k, k_*, \sigma) \right).$$

- And the GW spectrum is

$$\Omega_{\mathrm{GW}} = 6\mathcal{A}_{\mathcal{R}}^2 \frac{k^2}{2\pi\sigma^2} \left(\frac{k}{k_*}\right)^4 \int_0^\infty dv \int_{|1-v|}^{1+v} du\, uv\, \mathcal{T}(u, v)$$
$$\times \left[e^{-\frac{(vk-k_*)^2}{2\sigma^2}} + \sqrt{2\pi} F_{\mathrm{NL}}^2 \mathcal{A}_{\mathcal{R}} \frac{\sigma}{vk} \mathcal{F}(vk, k_*, \sigma) \right] \left[e^{-\frac{(uk-k_*)^2}{2\sigma^2}} + \sqrt{2\pi} F_{\mathrm{NL}}^2 \mathcal{A}_{\mathcal{R}} \frac{\sigma}{uk} \mathcal{F}(uk, k_*, \sigma) \right].$$

Induced GWs

- The result when σ<<k* is the integral (Cai, SP & Sasaki, 1810.11000):

$$\Omega_{\rm GW} = 6\mathcal{A}_{\mathcal{R}}^2 \frac{k^2}{2\pi\sigma^2}\left(\frac{k}{k_*}\right)^4 \int_0^\infty dv \int_{|1-v|}^{1+v} du\, uv\, \mathcal{T}(u,v)$$
$$\times \left[e^{-\frac{(vk-k_*)^2}{2\sigma^2}} + 2\mathcal{A}_{\mathcal{R}} F_{\rm NL}^2 \frac{\sigma}{vk}\sqrt{\frac{\pi}{2}}\,{\rm erf}\left(\frac{vk}{2\sigma}\right)\right]$$
$$\times \left[e^{-\frac{(uk-k_*)^2}{2\sigma^2}} + 2\mathcal{A}_{\mathcal{R}} F_{\rm NL}^2 \frac{\sigma}{uk}\sqrt{\frac{\pi}{2}}\,{\rm erf}\left(\frac{uk}{2\sigma}\right)\right].$$

$$\mathcal{T}(u,v) = \frac{1}{4}\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2\left(\frac{u^2+v^2-3}{2uv}\right)^2$$
$$\times \left\{\left(-2+\frac{u^2+v^2-3}{2uv}\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2\right.$$
$$\left.+\,\pi^2\left(\frac{u^2+v^2-3}{2uv}\right)^2\Theta\left(u+v-\sqrt{3}\right)\right\}.$$

Induced GWs

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Saito & Yokoyama, 0812.4339

$$\mathcal{T}(u,v) = \frac{1}{4}\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2\left(\frac{u^2+v^2-3}{2uv}\right)^2$$
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$$\left.+\,\pi^2\left(\frac{u^2+v^2-3}{2uv}\right)^2\Theta\left(u+v-\sqrt{3}\right)\right\}.$$

Kohri & Tareda, 1804.08577

Induced GWs

- The result when σ<<k* is the integral (Cai, SP & Sasaki, 1810.11000):

non-Gaussian contributions

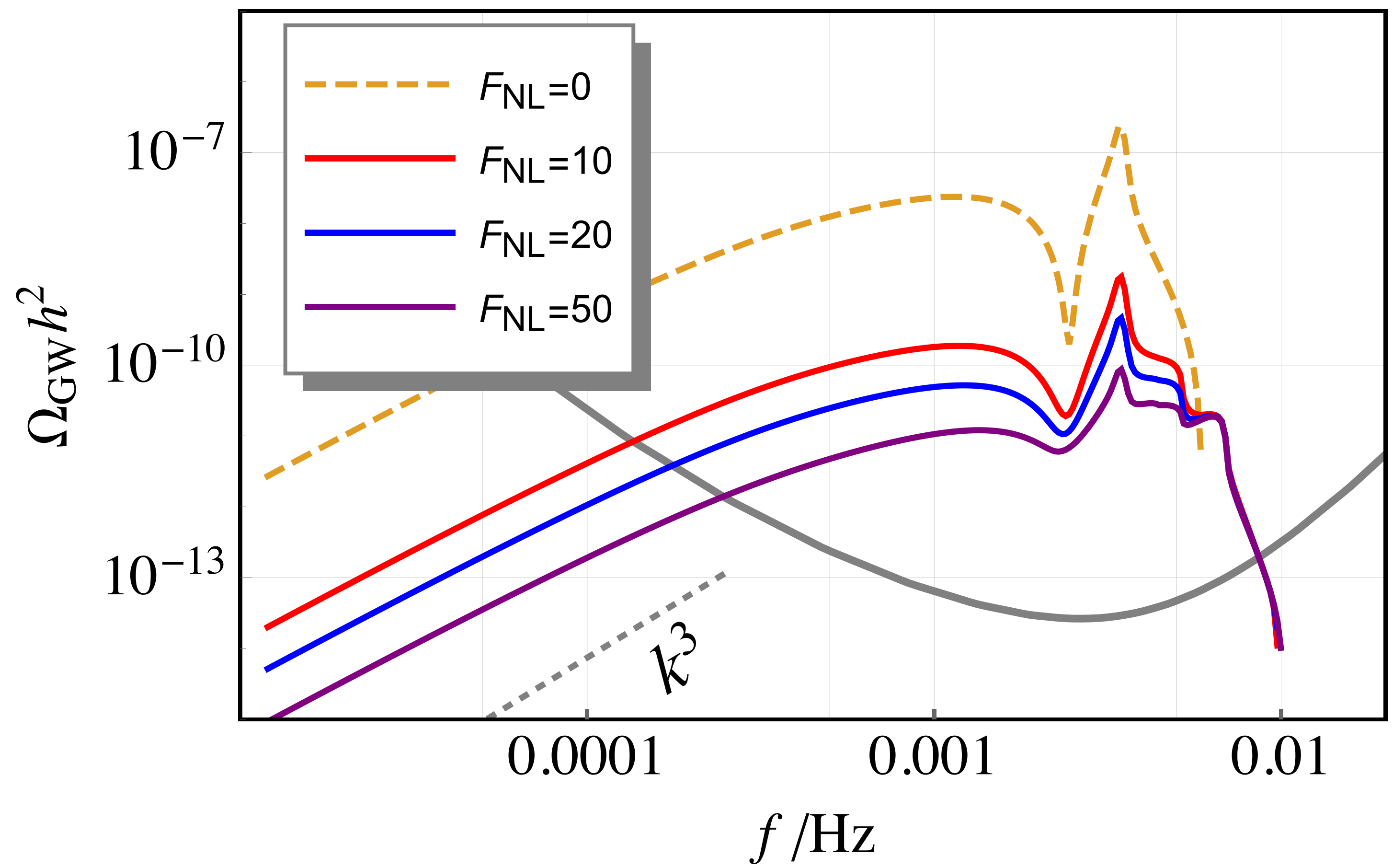
$$\Omega_{\rm GW} = 6\mathcal{A}_{\mathcal{R}}^2 \frac{k^2}{2\pi\sigma^2}\left(\frac{k}{k_*}\right)^4 \int_0^\infty dv \int_{|1-v|}^{1+v} du\, uv\, \mathcal{T}(u,v)$$
$$\times\left[e^{-\frac{(vk-k_*)^2}{2\sigma^2}} + 2\mathcal{A}_{\mathcal{R}}F_{\rm NL}^2 \frac{\sigma}{vk}\sqrt{\frac{\pi}{2}}\,{\rm erf}\left(\frac{vk}{2\sigma}\right)\right]$$
$$\times\left[e^{-\frac{(uk-k_*)^2}{2\sigma^2}} + 2\mathcal{A}_{\mathcal{R}}F_{\rm NL}^2 \frac{\sigma}{uk}\sqrt{\frac{\pi}{2}}\,{\rm erf}\left(\frac{uk}{2\sigma}\right)\right].$$

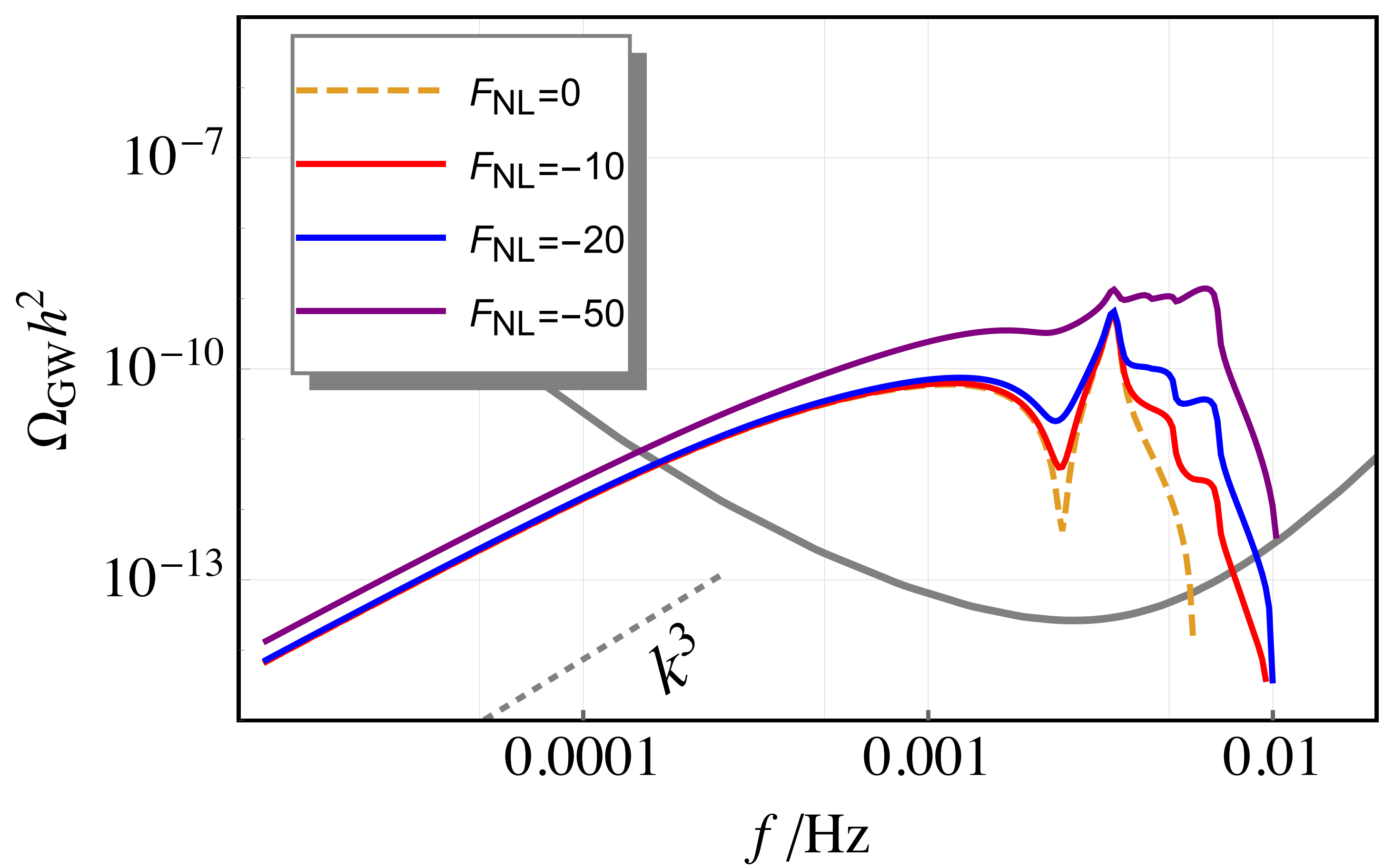
Saito & Yokoyama, 0812.4339

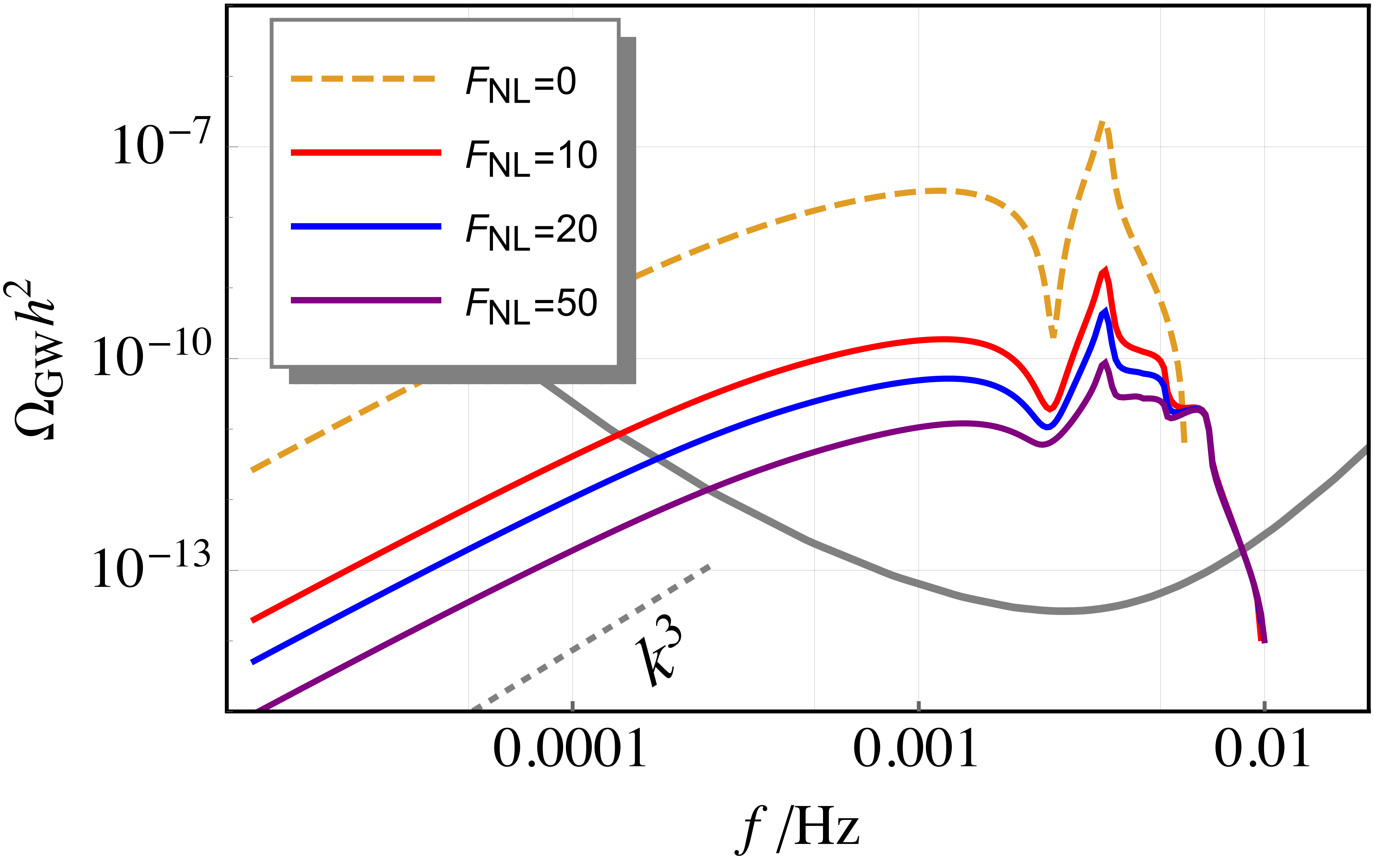
$$\mathcal{T}(u,v) = \frac{1}{4}\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2\left(\frac{u^2+v^2-3}{2uv}\right)^2$$
$$\times\left\{\left(-2+\frac{u^2+v^2-3}{2uv}\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2 + \pi^2\left(\frac{u^2+v^2-3}{2uv}\right)^2\Theta\left(u+v-\sqrt{3}\right)\right\}.$$

Kohri & Tareda, 1804.08577

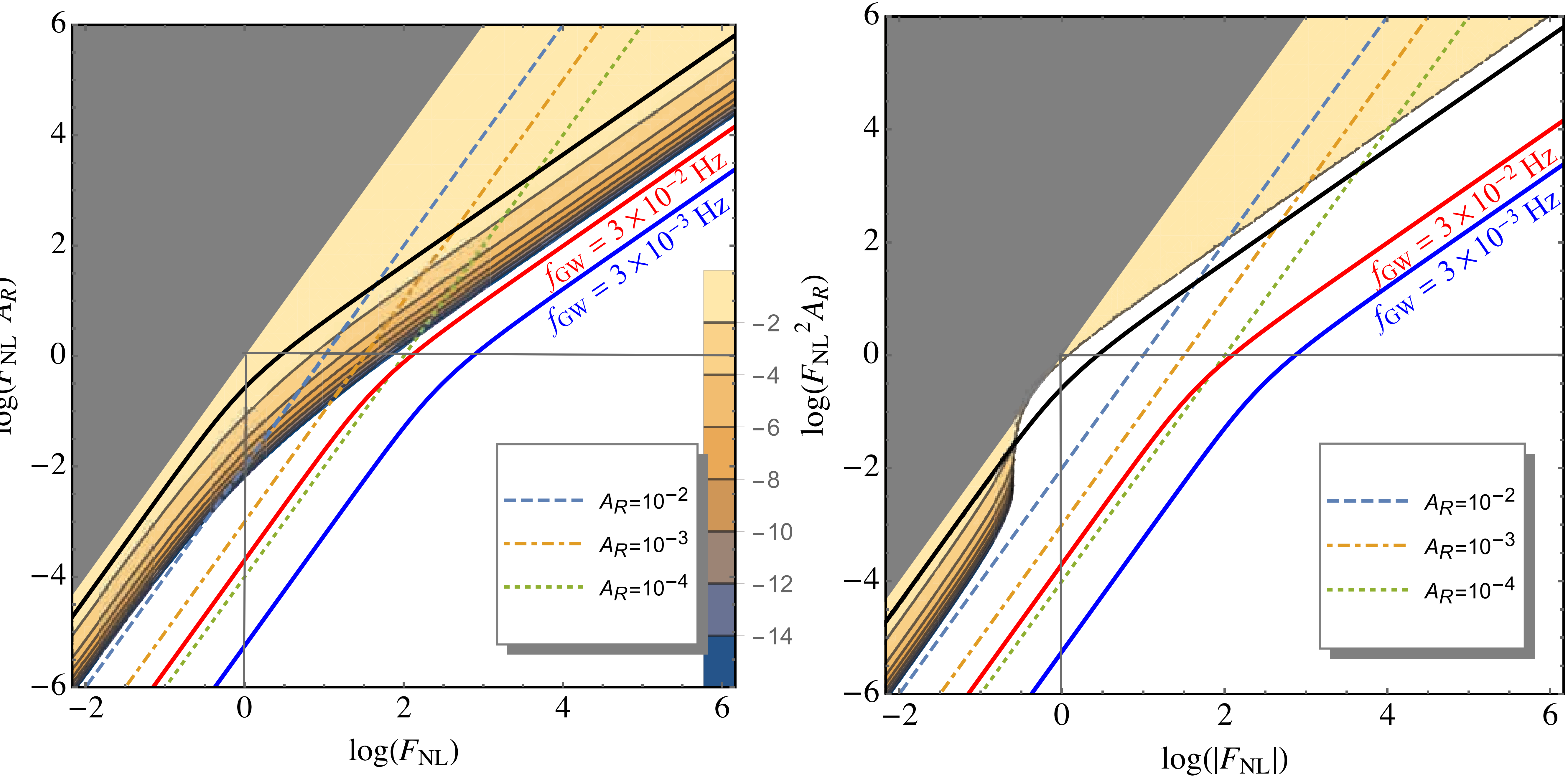
- Up: $F_{NL} > 0$, and we fix the PBH abundance to be 1.
- Down: $F_{NL} < 0$, and we fix the peak amplitude to be $\mathscr{A}_{\mathscr{R}} = 10^{-2}$
- Gray curve: LISA
- Frequency: PBH window <—> LISA band
- Coincidence, but fortunate for our universe.



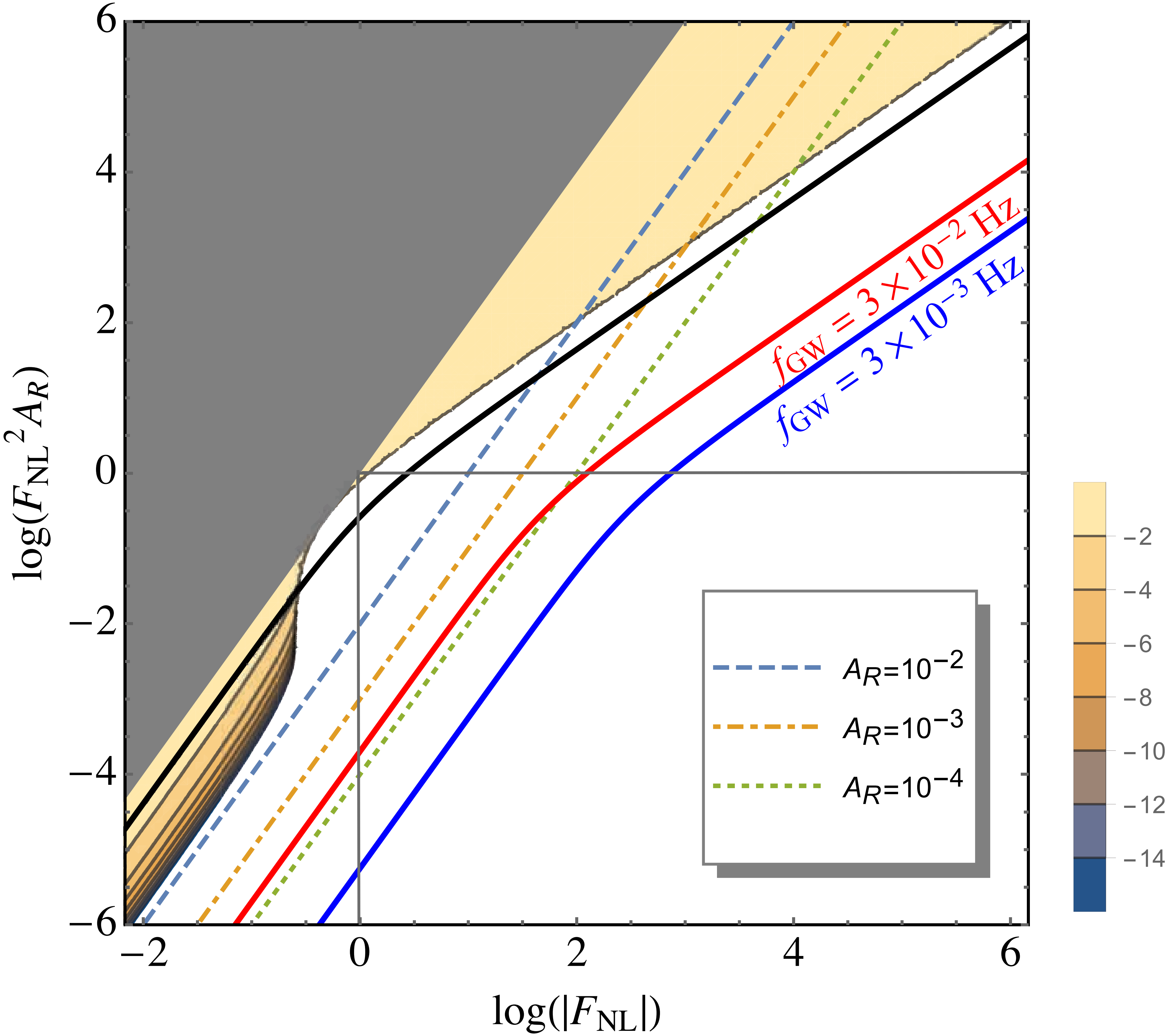




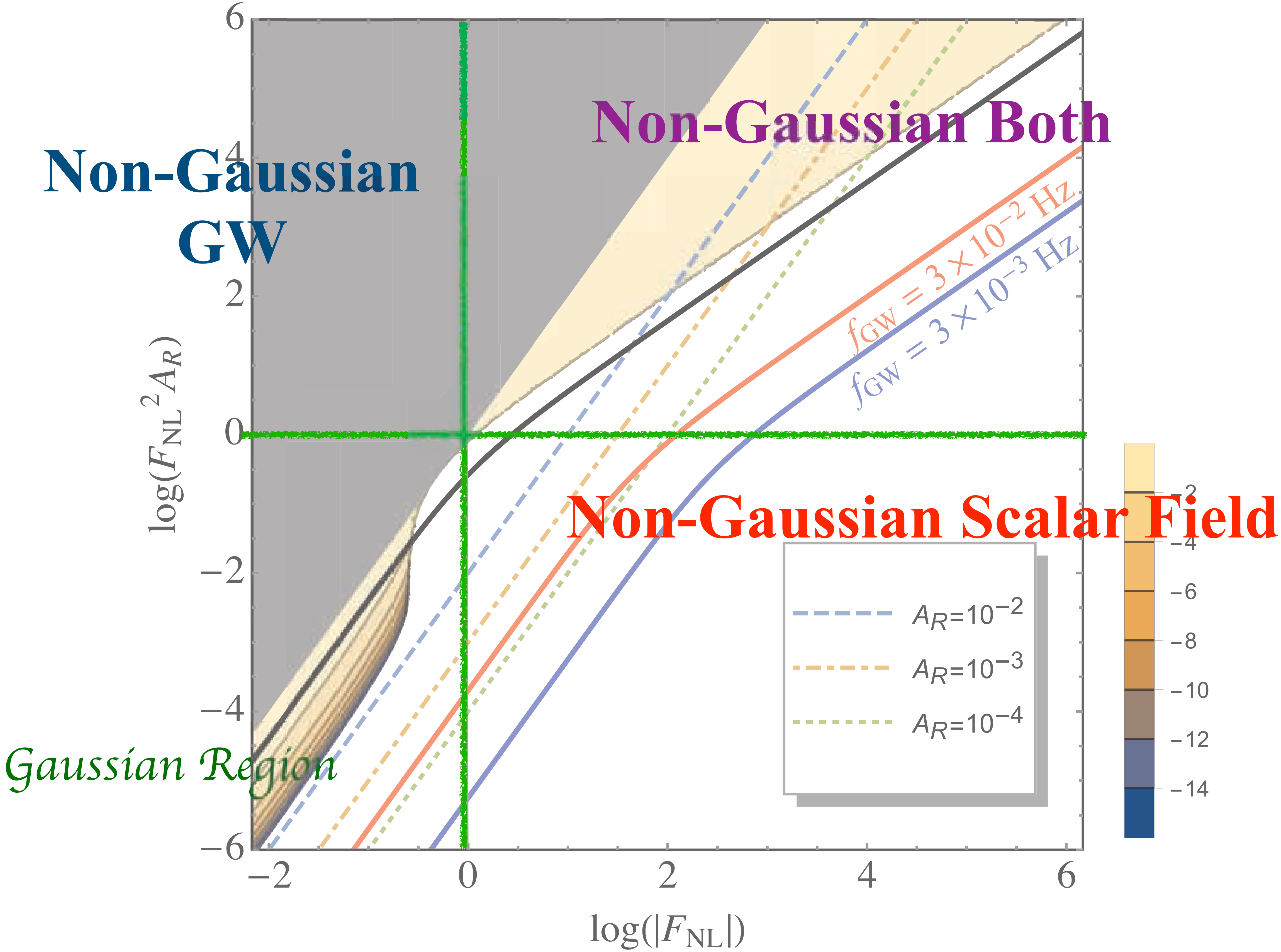
$F_{\rm NL}$=0
$F_{\rm NL}$=10
$F_{\rm NL}$=20
$F_{\rm NL}$=50
10^{-7}
10^{-10}
10^{-13}
$\Omega_{\rm GW}h^2$
k^3
0.0001
0.001
0.01
f /Hz



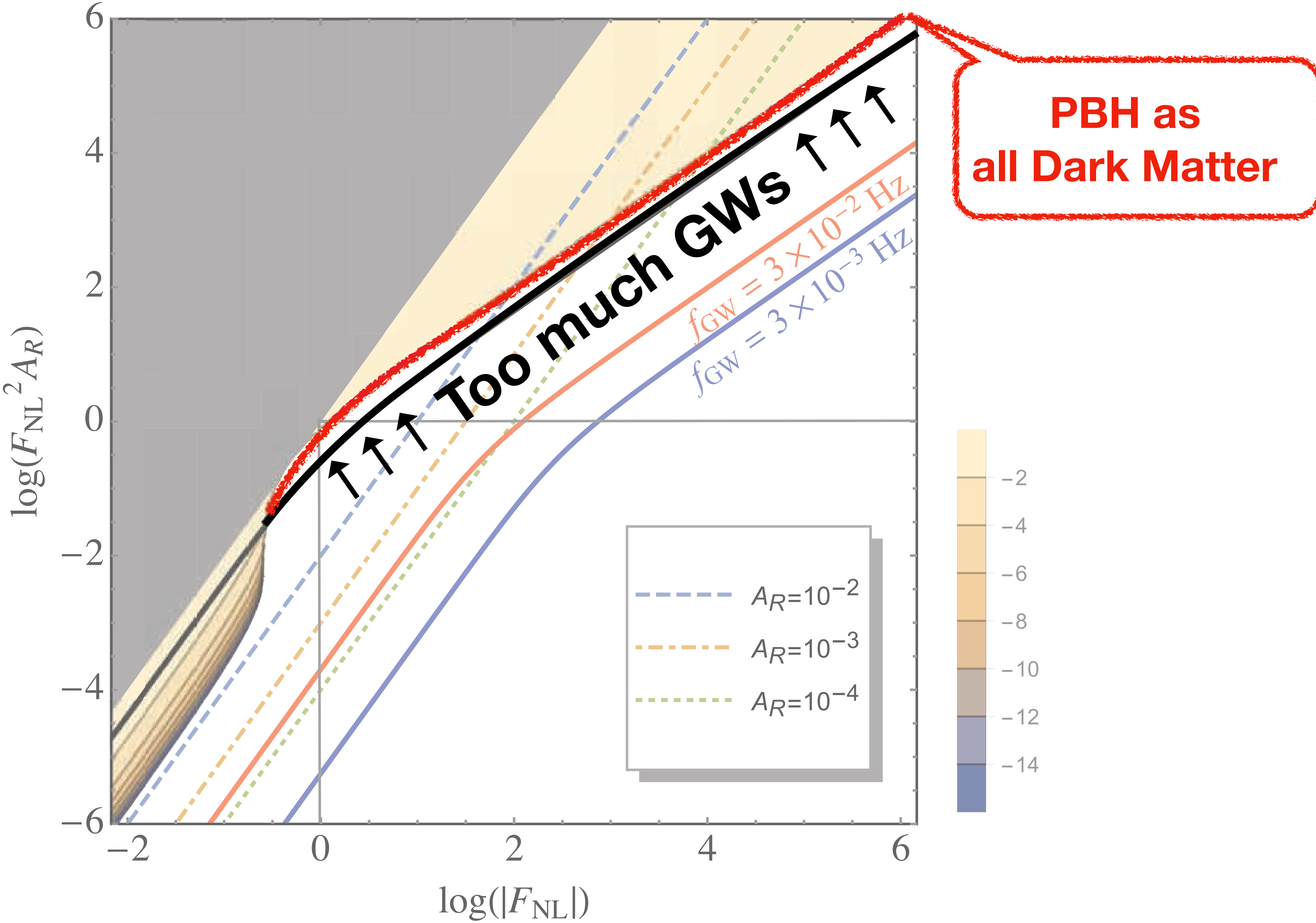
log(F_{NL})
log($|F_{NL}|$)
log($F_{NL}{}^2 A_R$)
$f_{GW} = 3\times10^{-2}$ Hz
$f_{GW} = 3\times10^{-3}$ Hz
A_R=10^{-2}
A_R=10^{-3}
A_R=10^{-4}
−2
−4
−6
−8
−10
−12
−14



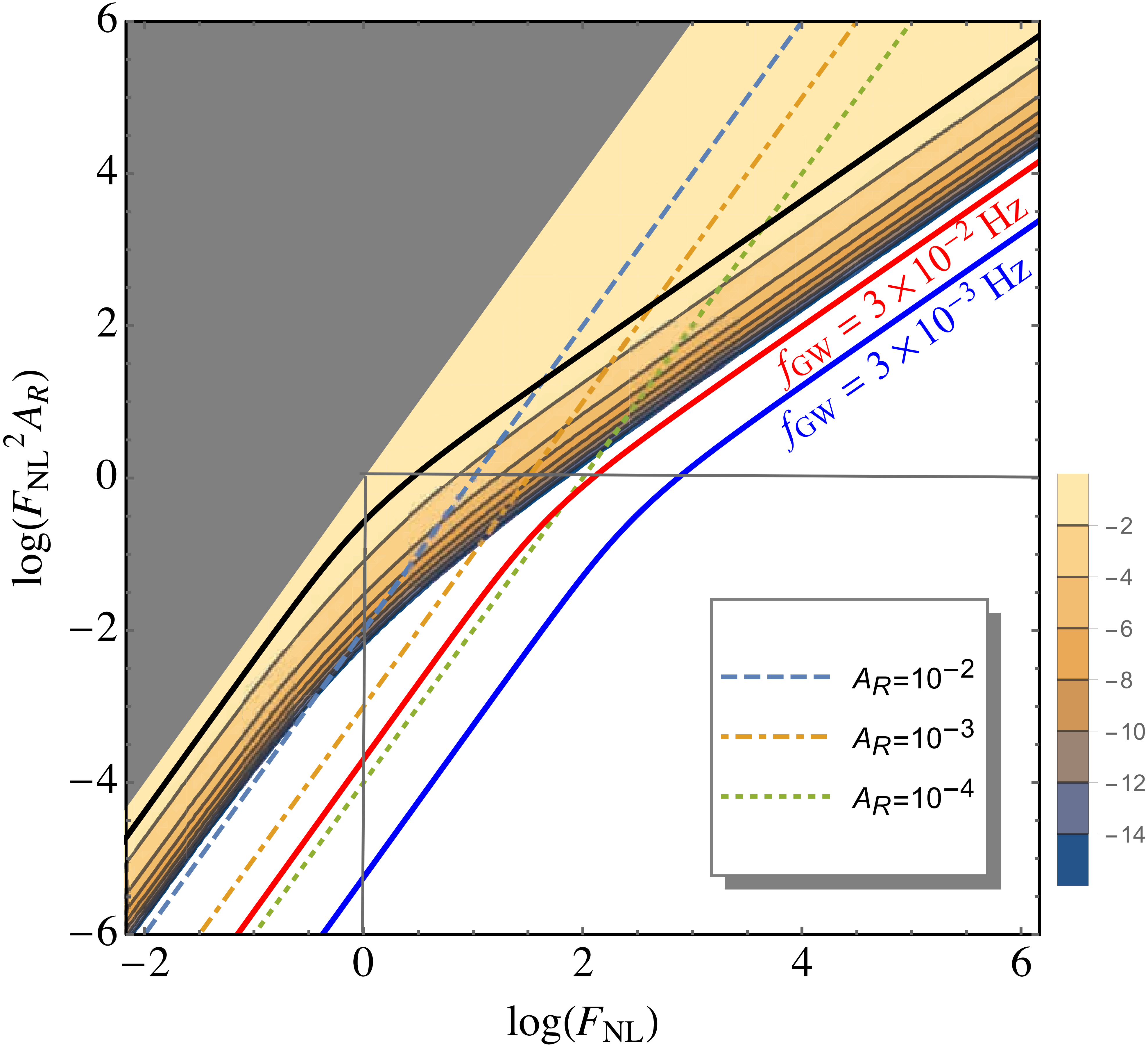
$\log(F_{\rm NL}{}^2 A_R)$
$\log(|F_{\rm NL}|)$
$f_{\rm GW} = 3\times10^{-2}$ Hz
$f_{\rm GW} = 3\times10^{-3}$ Hz
A_R=10^{-2}
A_R=10^{-3}
A_R=10^{-4}
−2
−4
−6
−8
−10
−12
−14



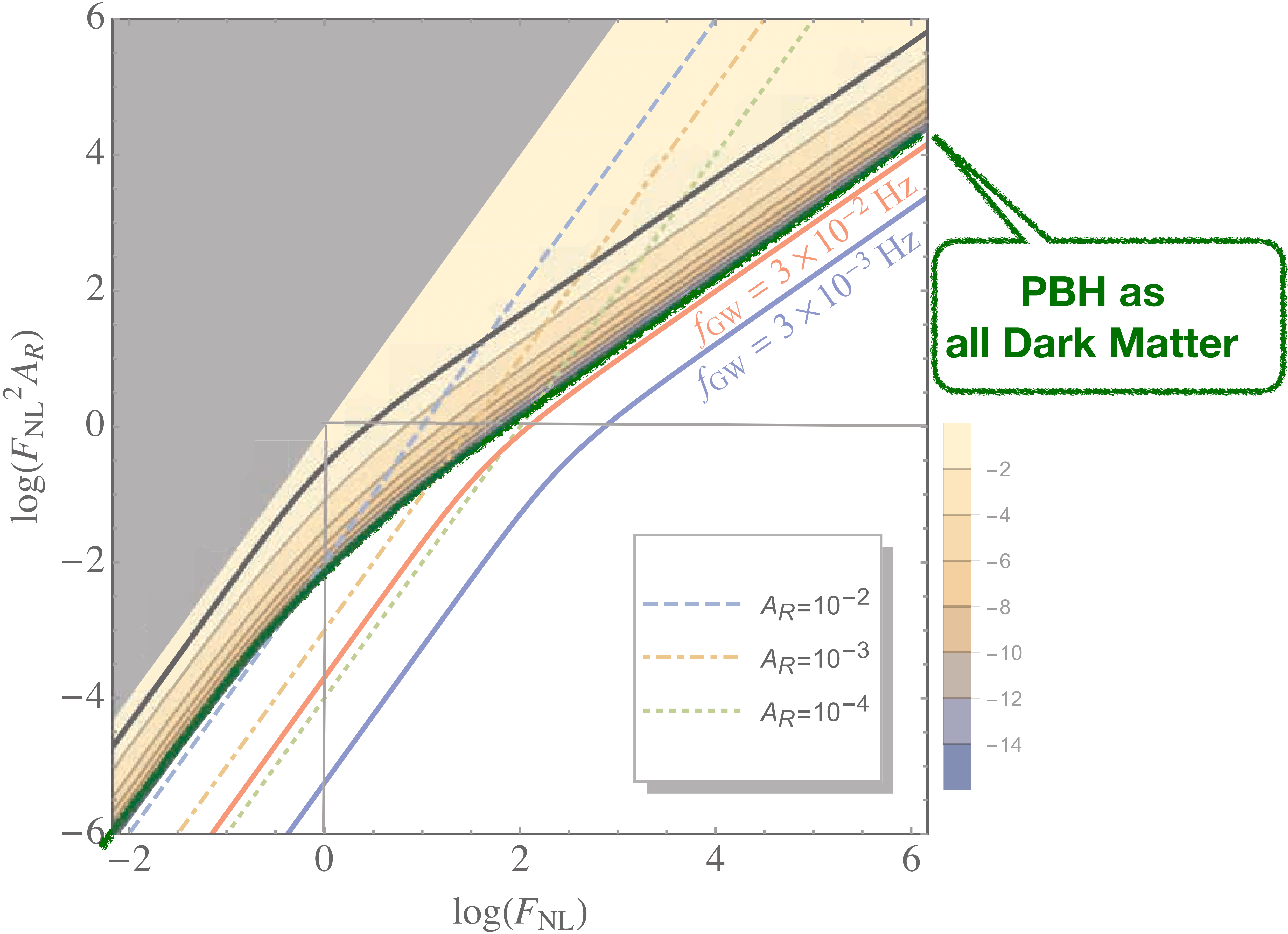
Non-Gaussian Both
Non-Gaussian GW
Non-Gaussian Scalar Field
Gaussian Region
$f_{\rm GW}=3\times10^{-2}$ Hz
$f_{\rm GW}=3\times10^{-3}$ Hz
$A_R=10^{-2}$
$A_R=10^{-3}$
$A_R=10^{-4}$
$\log(F_{\rm NL}{}^2 A_R)$
$\log(|F_{\rm NL}|)$
-2
-4
-6
-8
-10
-12
-14



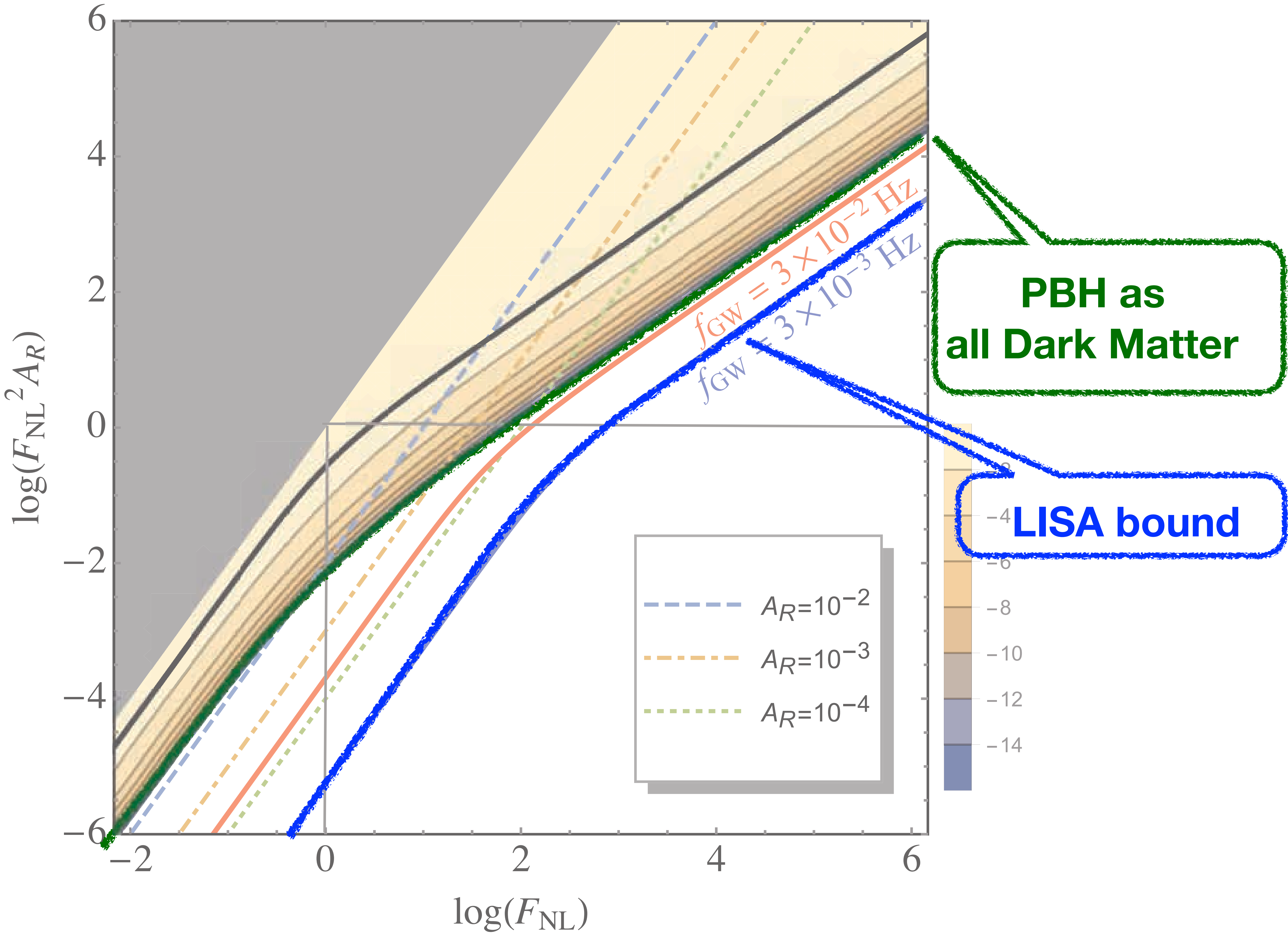
PBH as
all Dark Matter
Too much GWs
$f_{\rm GW} = 3\times10^{-2}$ Hz
$f_{\rm GW} = 3\times10^{-3}$ Hz
$A_R=10^{-2}$
$A_R=10^{-3}$
$A_R=10^{-4}$
$\log(F_{\rm NL}{}^2 A_R)$
$\log(|F_{\rm NL}|)$



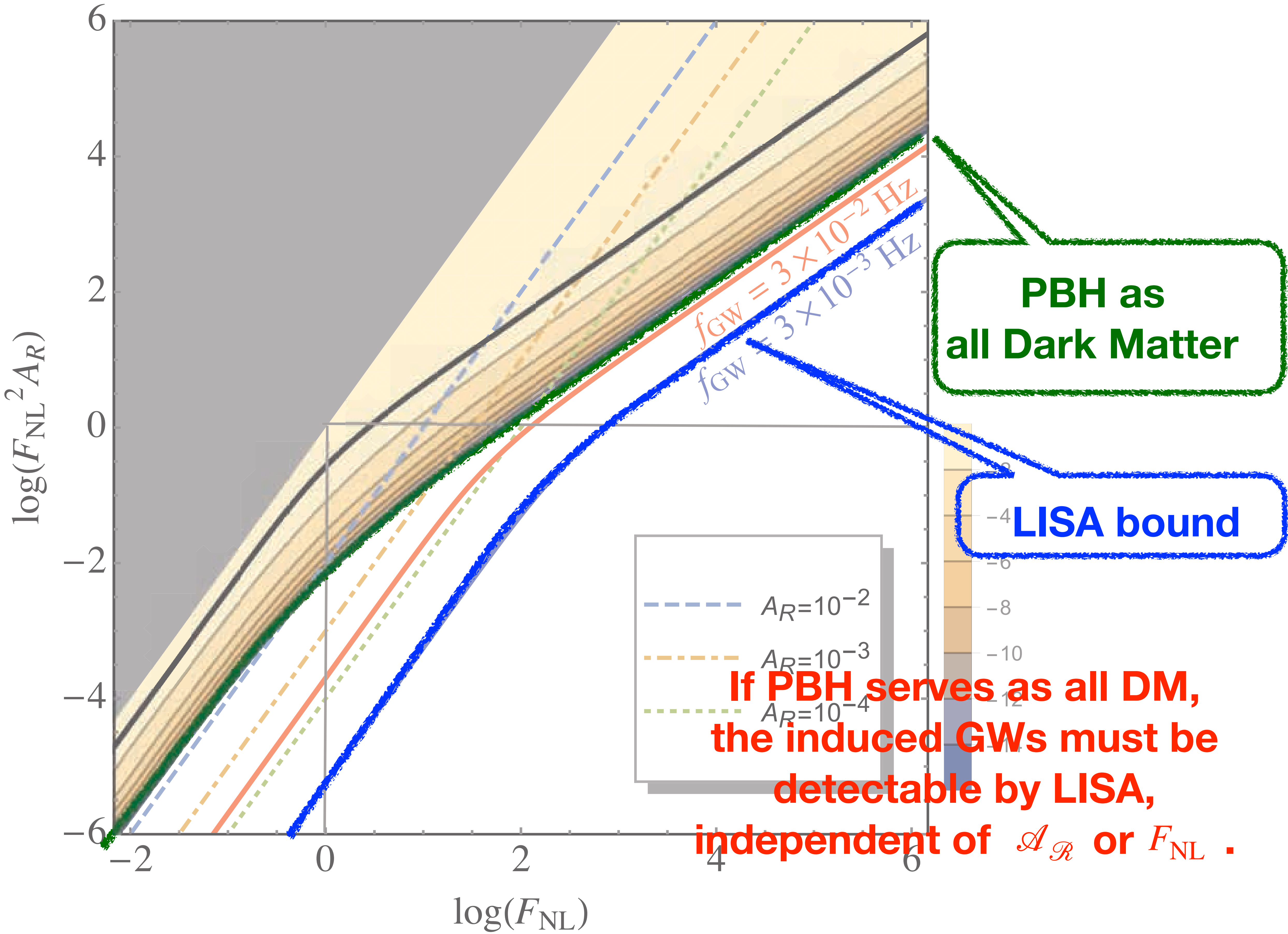
6
4
2
0
−2
−4
−6
log($F_{NL}{}^2 A_R$)
−2
0
2
4
6
log(F_{NL})
$f_{GW} = 3\times10^{-2}$ Hz
$f_{GW} = 3\times10^{-3}$ Hz
A_R=10^{-2}
A_R=10^{-3}
A_R=10^{-4}
−2
−4
−6
−8
−10
−12
−14



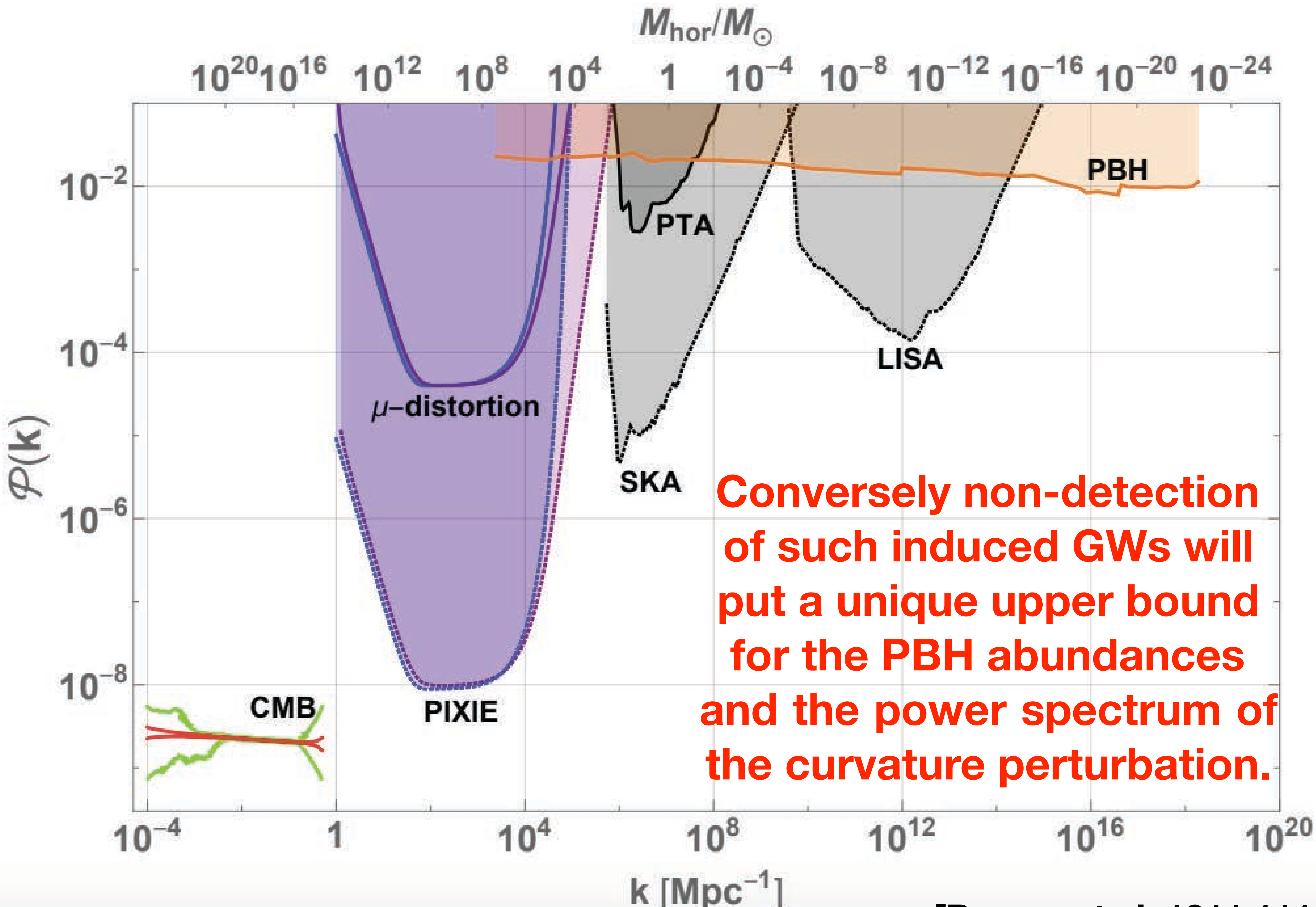
PBH as all Dark Matter
$f_{GW} = 3\times10^{-2}$ Hz
$f_{GW} = 3\times10^{-3}$ Hz
A_R=10^{-2}
A_R=10^{-3}
A_R=10^{-4}
$\log(F_{NL}{}^2 A_R)$
$\log(F_{NL})$



PBH as all Dark Matter
LISA bound
$f_{GW} = 3\times10^{-2}$ Hz
$f_{GW} = 3\times10^{-3}$ Hz
$A_R=10^{-2}$
$A_R=10^{-3}$
$A_R=10^{-4}$
−4
−6
−8
−10
−12
−14
$\log(F_{NL}{}^2 A_R)$
$\log(F_{NL})$
6
4
2
0
−2
−4
−6
−2
0
2
4
6



PBH as all Dark Matter
LISA bound
$f_{GW} = 3\times10^{-2}$ Hz
$f_{GW} = 3\times10^{-3}$ Hz
$A_R=10^{-2}$
$A_R=10^{-3}$
$A_R=10^{-4}$
$\log(F_{NL}{}^2 A_R)$
$\log(F_{NL})$
If PBH serves as all DM, the induced GWs must be detectable by LISA, independent of $\mathcal{A}_{\mathcal{R}}$ or $F_{\rm NL}$.



Conversely non-detection of such induced GWs will put a unique upper bound for the PBH abundances and the power spectrum of the curvature perturbation.

[Byrnes et al. 1811.11158]

Summary

- GWs induced by non-Gaussian scalar perturbations: k^3 - slope, multiple peaks, and a cutoff.
- If PBHs can serve as all the DM, induced GWs must be detectable by LISA, no matter how small $\mathscr{A}_{\mathscr{R}}$ or $f_{\rm NL}$ is.
- Conversely if LISA can not detect the induced GWs, we can put an independent constraint on the PBH abundances on mass range 10^{19}g to 10^{22}g where no current experiment can explore.

Thank you!